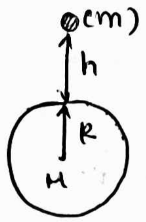
25/11/19

L-1/2

GRAVITATION

VARIATION DUE TO HEIGHT



$\Rightarrow \quad F = mg$

$\Rightarrow \quad g = \frac{GM}{R^2} \quad \Rightarrow \quad GM = gR^2$

$\Rightarrow \quad F = \frac{GMm}{(R+h)^2}$

$\Rightarrow \quad mg_h = \frac{GMm}{(R+h)^2}$

$\Rightarrow \quad \boxed{g_h = \frac{GM}{(R+h)^2}} \; / \; \boxed{g_h = \frac{gR^2}{(R+h)^2}}$

$\Rightarrow \quad g_h = \frac{g\cancel{R^2}}{\cancel{R^2}\left(1+\frac{h}{R}\right)^2}$

$\Rightarrow \quad g_h = g\left(1+\frac{h}{R}\right)^{-2}$

if $\quad \frac{h}{R} << 1$

$\Rightarrow \quad \boxed{g_h = g\left(1-\frac{2h}{R}\right)}$ For variatⁿ upto 5%.

Que. At what height g ↓es by 36% ?

sol. $g_h = 64\%$

$\Rightarrow \quad \frac{64}{100}g = g\frac{R^2}{(R+h)^2}$

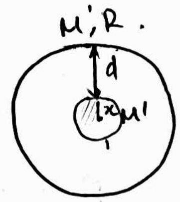
$\Rightarrow \quad \frac{R}{R+h} = \frac{8}{10}$

$\Rightarrow \quad 10R = 8R + 8h$

$2R = 8h$

$\Rightarrow \quad \boxed{R = 4h}$

VARIATION DUE TO DEPTH



$x = R - d$

$M' = \rho V'$

$\cancel{M' = \frac{4}{3}\pi R^3}$

$M' = \frac{M}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi (R-d)^3$

$$\boxed{M' = \frac{M(R-d)^3}{R^3}}$$

$F = \frac{GM'm}{R}$

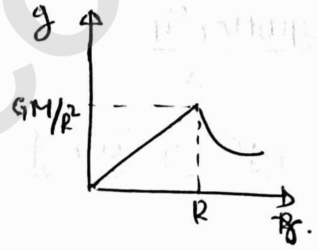
$\cancel{m} g_d = \frac{GM\cancel{m}}{(R-d)^2} \frac{(R-d)^3}{R^3}$

$g_d = \frac{GM(R-d)}{R^3}$

$g_d \Rightarrow \frac{gR^2 (R-d)}{R^3}$

$g_d = \frac{g(R-d)}{R}$

$$\boxed{g_d = g\left(1 - \frac{d}{R}\right)}$$



Que. At what Height value of g is equal to value of g at 50 m depth.

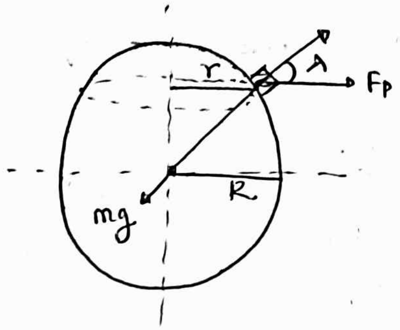
sol.

$\cancel{g_d = g(1 - 50}$

$1 - \frac{2h}{R} = 1 - \frac{d}{R}$

$$\boxed{d = 2h}$$

VARIATION IN g' due To rotatⁿ of Earth



$$mg' = mg - m\omega^2 r \cos^2\lambda$$

$$\boxed{g' = g - \omega^2 r \cos^2\lambda}$$

equator पे

$\lambda = 0$

$$\boxed{g' = g - \omega r}$$

Poles पे

$g' = g.$

Ques:- (M) $\Rightarrow$ (m) (M-m) are separated by distance r. for max Gravitational force. find ratio of $\frac{m}{M}$?

Sol.

$$F = \frac{G\, m(M-m)}{r^2}$$

for F_{max} $\boxed{\frac{dF}{dm} = 0}$

$$\frac{d\left[(M-m)m\right]}{dm} = 0$$

$$M - 2m = 0$$

$$\boxed{\frac{m}{M} = \frac{1}{2}}$$

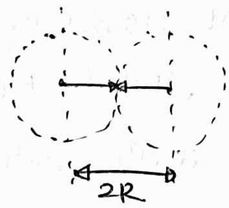
Ques.:- Two identical solid (M, R) approach each other & collide find

(A) Acc. of each sphere just before collision.

(B) force of attraction b/w them if they are kept in contact.

Sol. (A)

$$F = \frac{GM^2}{4R^2}$$



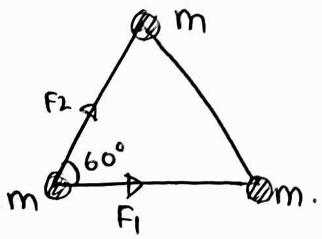
(B)

$$a = \frac{F}{M} = \frac{GM}{4R^2}$$

$$a_{rel} = 2a = \frac{2GM}{4R^2} = \frac{GM}{2R^2}$$

Ques. 3 identical point masses M each are kept at the vertices of an equilateral triangle of side length 'a'. F_g on one of mass?

Sol.



$$F_1 = \frac{GM^2}{a^2}$$

$$F_0 = \sqrt{3}F$$

$$\Rightarrow \sqrt{3}\frac{GM^2}{a^2}$$

Ques. 2 point of objects having diff mass attract each other with Gravitational force. If force one one of them is $\vec{F_1} = a\hat{i} - b\hat{j} - c\hat{k}$. Find force $\vec{F_2}$ on the other. what is Magnitude of $\vec{F_2}$?

Sol.

$$F_1 = F_2$$

$$\vec{F_1} + \vec{F_2} = 0. \quad \text{(due to mutual interactⁿ)}$$

$$\vec{F_1} = -\vec{F_2}$$

$$\vec{F_2} = -a\hat{i} + b\hat{j} + c\hat{k}$$

Ques. Relation b/w. inertial & Gravitational Mass?

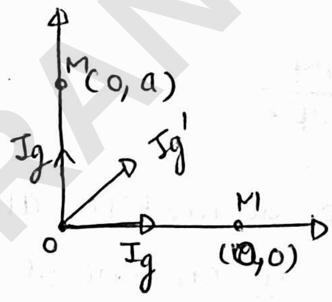
Sol.

$$m_i = \frac{F}{a} \qquad m_g = \frac{F}{g}$$

$$\boxed{\frac{m_i}{m_g} = 1}$$

Ques. Two point Masses M are placed at points having co-ordinate (a,0) or (0,a). Find Magnitude & directⁿ of resultant Gravitational Field at origin. write field in vector notatⁿ.

Sol.



$$\vec{E} = \frac{GM}{a^2}\hat{i} + \frac{GM}{a^2}\hat{j}.$$

$$\vec{E} = \frac{GM}{a^2}(\hat{i} + \hat{j}).$$

$$I_g' = \sqrt{2} I_g$$

$$= \sqrt{2}\frac{GM}{a^2}.$$

Ques. How Gravitational field on surface of a spherical ~~surface~~ planet depends on its density (ρ) & Radius(R)?

Sol.

$$g = \frac{GM}{R^2} \Rightarrow g \propto \frac{M}{R^2}$$

$$= \frac{G}{R^2} \frac{4}{3}\pi R^3 \rho$$

$$g = \frac{4}{3}\pi G (\rho R)$$

$$\boxed{g \propto \rho R}$$

Ques. If g_s is the gravitational acc. of surface of earth. find the same at

i) At a point $R/2$ above ii) $\frac{R}{2}$ below the surface of earth.

$$\boxed{g_s = \frac{GM}{R^2}}$$

i) $R' = R + \frac{R}{2} = \frac{3R}{2}$

ii) $R' =$

i) $g_h = \frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{4}{9}\left(\frac{GM}{R^2}\right) = \frac{4}{9} g_s.$

ii) $g_d = \frac{GM}{R^3}(R-d) = \frac{GM}{R^3} \times \frac{R}{2} = \frac{GM}{R^2} \times \frac{1}{2} = \frac{1}{2} g_s.$

Ques. At what Height above the surface of earth (radius R) weight of an object becomes half that on the surface.
Also at what ~~Height~~ depth below the surface of earth it will have same as g_h.

Sol. i)

$$\frac{g_s}{g_h} = \frac{R^2}{(R+h)^2} \Rightarrow \frac{1}{2} = \frac{R^2}{(R+h)^2}.$$

$$\Rightarrow \frac{R^2}{(R+h)^2} = \frac{1}{2}$$

$$\Rightarrow \frac{R}{R+h} = \frac{1}{\sqrt{2}}.$$

$$\Rightarrow \sqrt{2}R = R+h.$$

$$\Rightarrow \boxed{h = (\sqrt{2}-1)R}$$

ii)

$$\frac{1}{2} = 1 - \frac{d}{R} \Rightarrow \frac{d}{R} = \frac{1}{2} \Rightarrow \boxed{d = \frac{R}{2}}$$

Ques $M_e = M$ $\quad M_o = 2M$

$g_e = g$ $\quad R_o = \frac{R}{2}$

$R_e = R$ $\quad g_o = ?$

Sol.

$$g \propto \frac{M}{R^2}$$

$$\frac{g_e}{g_o} = \frac{M_e}{R_e^2} \times \frac{R_o^2}{M_o}$$

$$\frac{g}{g_o} = \frac{\cancel{M}}{\cancel{R^2}} \times \frac{R^2}{2M\,8}$$

$$\boxed{g_o = 8g}$$

Ques.

$$\frac{g_A}{g_B} = \frac{4}{1} \qquad \frac{r_A}{r_B} = \frac{16}{1}$$

find $\frac{\rho_A}{\rho_B} = ?$

Sol.

$$\boxed{g \propto \rho R}$$

$$\frac{g_A}{g_B} = \frac{\rho_A}{\rho_B} \times \frac{R_A}{R_B}$$

$$\frac{4}{1} = \frac{\rho_A}{\rho_B} \times \frac{16}{1}$$

$$\boxed{\frac{\rho_A}{\rho_B} = \frac{1}{4}}$$

Ques. A boy can throw a ball upto 18m on earth surface. what will be value of Maximum Height he can throw. on surface of spherical planet having $\frac{3}{2}$ times. density & double Radius as compared to earth.

Sol.

$$\boxed{g \propto \rho R}$$

$$h = \frac{u^2}{2g} = \frac{u^2}{2\rho R}$$

$$\boxed{h \propto \frac{1}{\rho R}}$$

$$\frac{h}{h'} = \frac{1.5\rho \times 2R}{\rho \times R.}$$

$$\boxed{h' = \frac{h}{3}} \qquad \therefore h' = \frac{18}{3} = 6m.$$

VARIATION DUE TO SHAPE/size

* $R_p < R_{eq}$.
* $R_{eq} = R_p + 21\,km$.

$$g_p = \frac{GM}{R_p^2}$$

$$g_{eq} = \frac{GM}{R_{eq}^2} \Rightarrow g_p > g_{eq}.$$

*

Ques. from 2 shops located At pole and equator respectively → The customer will get more mass of sugar at equator.

$$W_1 = W_2.$$

$$m_1 g_1 = m_2 g_2.$$

$$\frac{m_1}{m_2} = \frac{g_2}{g_1}$$

$$\frac{m_p}{m_{eq}} = \frac{g_{eq}}{g_p} < 1$$

$$\boxed{m_p < m_{eq}}$$

Ques. * Difference in acc. due to gravity at poles + equator.

$$g_p - g_{eq} = 0.05\,m/s^2$$

Rotation $= 0.03\,m/s^2$

shape $= 0.02\,m/s^2$

Ques. if earth stops rotating suddenly then weight of body will ↑es everywhere on earth. (True / False)

sol.

True

$$g' = g - \omega^2 R\cos^2\lambda$$

at poles / At $\omega = 0$

$$g' = g.$$

∴ mass will ↑es

False.

At poles $\lambda = 90°$

$g' = g$ (independent of rotat^n)

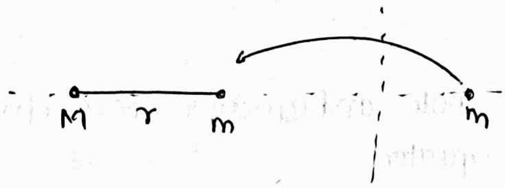
Always constant at poles.

* when rotatn of earth stops suddenly then weight of body will ↑es everywhere except poles.

GRAVITATIONAL POTENTIAL ENERGY

Infinity :- field से बाहर के सारे point को inf

* It is work done in bringing a mass 'm' from infinity to desired point to configure the system without changing the K.E.



$$W_{ext} = -W_{sys.} = -\int \vec{F}_{cons.} \cdot \vec{ds} = -\int_{\infty}^{r} \frac{GMm}{r^2} \cdot dr = -\frac{GMm}{r}$$

$$W_{ext} = -W_{sys} = -\frac{GMm}{r}$$

$$\Delta U = -\frac{GMm}{r} = U_p - U_\infty$$

$$\boxed{U_p = -\frac{GMm}{r} + U_\infty}$$

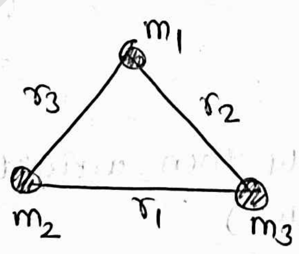
$U_\infty = 0$ (Reference)

$$\boxed{U_p = -\frac{GMm}{r}}$$ → At surface of earth.

At a ht. H

$$U = -\frac{GMm}{R+h}$$

TYPES OF QUESTIONS FORMED



(1) P.E of system

= W.D to configure the system.

$$= 0 + \left(-\frac{Gm_1m_2}{r_3}\right) + \left(-\frac{Gm_2m_3}{r_2}\right) + \left(-\frac{Gm_3m_2}{r_1}\right)$$

$$= -G\left[\frac{m_1m_2}{r_3} + \frac{m_2m_3}{r_1} + \frac{m_1m_3}{r_2}\right]$$

② **W.D to break the system**

$$= -[\text{P.E of system}]$$

$$= \frac{Gm_1m_2}{r_3} + \frac{Gm_2m_3}{r_1} + \frac{Gm_1m_3}{r_2}.$$

③ **Energy of m_1**

$$\boxed{U_1 = \left(-\frac{Gm_1m_2}{r_3}\right) + \left(-\frac{Gm_1m_3}{r_2}\right)}$$

④ Energy Required to make escape 'm_1' from the system.

= escape energy of m_1

= Always in form of K.E

$$= (-\text{PE of } m_1) = Gm_1\left[\frac{m_2}{r_3} + \frac{m_3}{r_2}\right]$$

⑤ **ESCAPE VELOCITY** (escape spd. of m_1)

$$\frac{1}{2}mV_e^2 = Gm_1\left[\frac{m_2}{r_3} + \frac{m_3}{r_2}\right]$$

$$\boxed{V_e = \sqrt{2Gm_1\left[\frac{m_2}{r_3} + \frac{m_3}{r_2}\right]}}$$

Que.

$$\text{P.E of system} = -\frac{GMm}{R}$$

$$\text{P.E of 'm'} = -\frac{GMm}{R}$$

energy req. to escape 'm'

$$= +\frac{GMm}{R}$$

* escape spd. of 'm'

$$\frac{1}{2}\cancel{m}V_e^2 = \frac{GM\cancel{m}}{R}$$

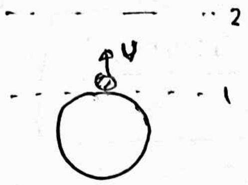
$$\boxed{V_e = \sqrt{\frac{2GM}{R}}}$$

6/12/19

Question based on conservat^n of Mech. Energy

Que. With what velocity A particle is thrown from surface of the earth to reach max. height $= h$

Sol.



$$K_i + U_1 = \cancel{K_2}^{0} + U_2$$

$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0 + \left(-\frac{GMm}{R+h}\right)$$

$$\Rightarrow \frac{v^2}{2} = \frac{GM}{R} - \frac{GM}{R+h}$$

$$\Rightarrow \frac{v^2}{2} = GM\left[\frac{h}{R(R+h)}\right]$$

$$\boxed{v = \sqrt{\frac{2gh}{\left(1+\frac{h}{R}\right)}}}$$

$h \ll$; $g' = g\left(1-\frac{2h}{R}\right)$

$\frac{1}{2}mv^2 = mgh$

$v^2 = \boxed{2hg\left(1-\frac{2h}{R}\right)}$

Que. find Height attained by body if it is projected by velocity 'v'?

$$K_1 + U_1 = \cancel{K_2}^{0} + U_2.$$

$$\Rightarrow \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0 + \left[-\frac{GMm}{R+h}\right].$$

$$\Rightarrow \frac{1}{2}\cancel{m}v^2/2$$

$$\Rightarrow \frac{v^2}{2} - \frac{GM}{R} = -\frac{GM}{R+h}$$

$$\Rightarrow \frac{GM}{R+h} = \frac{GM}{R} - \frac{v^2}{2}$$

$$\Rightarrow \frac{v^2}{2} = \frac{GMh}{(R+h)R}$$

$$\Rightarrow v_s^2 = \frac{v_e^2 h}{R+h}$$

$$\Rightarrow v_s^2(R+h) = v_e^2 h.$$

$$\Rightarrow \boxed{h = \frac{Rv^2}{v_e^2 - v^2}}$$

$K_i + U_i = K_f + U_f$

$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+h)}$

$\frac{1}{2}mv^2 = GM$

$v^2 = \frac{2gh}{(R+h)}$

$\frac{v^2}{2g}(R+h) = h$

$h =$

$\frac{GM}{R+h} - \frac{GM}{}$

$\frac{v^2R}{2g} + \frac{v^2h}{2g} = h$

$\frac{v^2R}{2g} = h - \frac{v^2h}{2g}$

$h = \frac{v^2R}{2g} \times \frac{2g}{1-v^2}$ $h = v^2R$

Que. find the escape spd. of a particle from the surface of the earth.

Sol.

- - - - ∞ (2)

$$K_1 + U_1 = K_2 + U_2.$$

- - - - - - (1)

$$K_s + U_s = K_\infty + \cancel{U_\infty} \to 0 \text{ always.}$$

$K_\infty \to = 0$ if K_s is min.

$$\frac{1}{2}mv^2 + \left[-\frac{GMm}{R}\right] = 0.$$

$$\frac{v^2}{2} = \frac{GM}{R}.$$

$$\boxed{V_e = \sqrt{\frac{2GM}{R}}}$$

Que. If spd. given is more than require ~~spd~~ escape spd the find v_f.

Sol. Here $K_\infty \neq 0$.

means

$$K_\infty = \frac{1}{2}mv_\infty^2.$$

$$K_s + U_s = K_\infty + \cancel{U_\infty} \to 0.$$

$$\frac{1}{2}mv_s^2 + \left(-\frac{GMm}{R}\right) = \frac{1}{2}mv_\infty^2.$$

$$v_\infty^2 = v_s^2 - v_e^2 \qquad V_e = \sqrt{\frac{2GM}{R}}.$$

$$\boxed{V_\infty = \sqrt{V_s^2 - V_e^2}}$$

"Spd. at ∞" / "interstellar spd"

ESCAPE SPEED OF EARTH :-
(from surface)

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = 11.2 \text{ km/s}$$

* Depends on mass & Radius of earth.
* Does not depend on mass of the object + dircn of projectn.
* Main reason for atmosphere on a Planet (Earth)

⇒ $\boxed{V_{rms} \text{ of gas molecules} < V_e}$

GRAVITATIONAL FIELD

→ It is an imaginary field of influence around a mass upto which a particle can exert the force on another mass.

* Gravitational Field Intensity / $\vec{E}$

→ it is a vector physical quantity i.e used to determine the strength of Gravitational field at a particular point due to the mass.

→ Defined as. force per unit mass.

$\vec{r} = \vec{OP}$

P पर M के कारण G.F.I = ?

$$\vec{I_g} = \frac{\vec{F}}{m} = -\frac{GM}{r^2} (\hat{OP})$$

"–ve" ⇒ dircn towards the mass.

vector form $\vec{I_g} = \frac{GM}{r^2}(-\hat{r})$

$$\boxed{\vec{I_g} = -\frac{GM}{r^3}\vec{r}}$$

~~unit → kg/m~~ unit ⇒ N/kg.

GRAVITATIONAL POTENTIAL :- (V).

* scalar physical Quantity
* Type of work done.
*

'p' पर M के कारण Grav. Potential =?

$$V_p - V_\infty = \frac{U_p - U_\infty}{m}$$

$$\boxed{\Delta V = \frac{\Delta U}{m}}$$

$$\boxed{V_p = -\frac{GM}{R}}$$

coz potential energy is -ve here

* W.D in taking unit mass from ∞ to desired point without changing its K.E, against grav. field of mass M.

IMP RELATIONS :-

* $\vec{I_g} = \frac{\vec{F}}{m}$

$$\boxed{\vec{F} = m\vec{I_g}}$$

* $V_p = \frac{U_p}{m}$

$$\boxed{U_p = mV_p}$$

for a const. force.

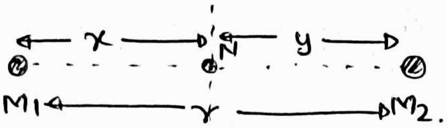
$$\left.\begin{aligned} F &= -\frac{dU}{dr} \\ dU &= -\vec{F}\cdot\vec{dr} \end{aligned}\right\} \Rightarrow$$

$$m.dV = -m\vec{I_g}\cdot\vec{dr}$$

$$dV = -\vec{I_g}\cdot\vec{dr}$$

$$\boxed{\Delta V = -\int \vec{I_g}\cdot\vec{dr}}$$

CONCEPT OF NEUTRAL POINT (same as that in $\vec{E}$).

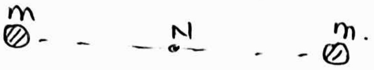


'N' पर

$$\left[\begin{array}{ll} \vec{F}=0 & \text{on 3rd particle} \\ \vec{I_g}=0 & \text{(Resultant)} \end{array}\right]$$

$$\boxed{x = \frac{r\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}} \qquad \boxed{y = \frac{r\sqrt{M_2}}{\sqrt{M_1}+\sqrt{M_2}}}$$

Ques.



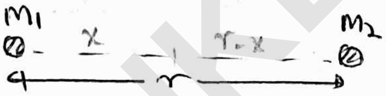
find Neutral point का potential = ?

Sol.

$$V_N = -\frac{GM}{r/2} - \frac{GM}{r/2}$$

$$\boxed{V_N = -\frac{4GM}{r}}$$

Ques.

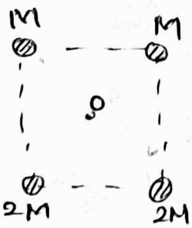


Neutral point का potential = ?

$$V_N = -\frac{GM_1}{x} - \frac{GM_2}{\cancel{y}}$$

$$\boxed{V_N = -\frac{G}{r}(\sqrt{m_1}+\sqrt{M_2})^2}$$

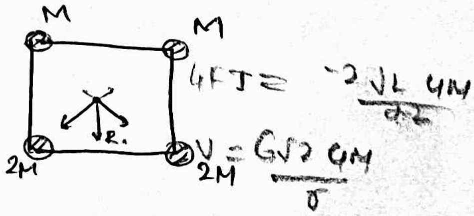
Ques.



centre of square O. O पर GFI and potential

Sol. GFI

$$I_g = -\frac{GM}{r^2}.$$

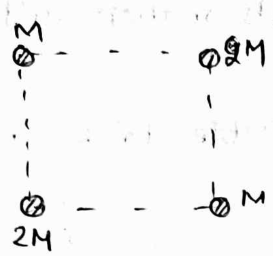


$$\Rightarrow \left[-\frac{2GM}{(a/\sqrt{2})^2}\right] - \left[-\frac{GM}{(a/\sqrt{2})^2}\right]$$

$$\Rightarrow \quad -\frac{4GM}{a^2} + \frac{2GM}{a^2} = -\frac{2GM}{a^2} \Rightarrow -\frac{2GM\sqrt{2}}{a^2}$$

$$V = -\frac{GM}{r} \Rightarrow -\frac{6GM}{r} = -\frac{6GM}{a\sqrt{2}} = -\frac{6\sqrt{2}GM}{a}$$

~~Ques.~~



Sol.

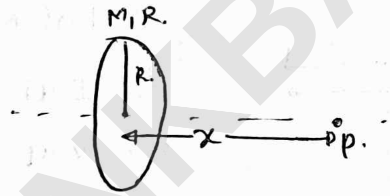
$I_g = 0$

$$V = -\frac{GM}{r} = -\frac{6GM}{r}$$

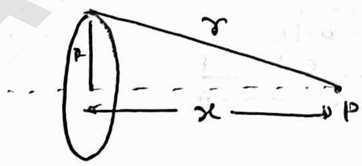
$$= -\frac{6GM}{a/\sqrt{2}} = -\frac{6\sqrt{2}GM}{a}$$

Ques. Due to ring of Mass = M (at a axial distance = x)
Radius = R

~~Sol.~~



Sol.



$$\boxed{I_g = \frac{-GMx}{(R^2+x^2)^{3/2}}}$$

$$\boxed{V = \frac{-GM}{\sqrt{x^2+R^2}}}$$

* center पर $x = 0$

$I = 0$

$$\boxed{V = -\frac{GM}{R}}$$

* $x \ll R$

$$\boxed{I = -\frac{GMx}{R^3}}$$

$$\boxed{V = -\frac{GM}{R}}$$

* $x \gg R$ (very far point)

- $I = -\frac{GM}{x^2}$
- $V = -\frac{GM}{x}$

} behave like a particle.

** P पर m रख दिया जाये तो

$$\boxed{F = \frac{-GMmx}{(x^2+R^2)^{3/2}}} \qquad \boxed{U = \frac{-GMm}{\sqrt{R^2+x^2}}}$$

Quer. Particle of mass = m is released from P, then its motion will be.

Sol.

⇒ if nothing is mentioned or 'x' is comparable to R the "OSCILLATORY"

⇒ if $x << R$ is mentioned ⇒ SHM

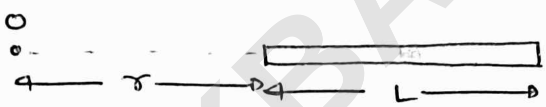
Quer. Find Time period of SHM (if $x << R$).?

$$F = -\frac{GMmx}{R^3} = -m\omega^2 x.$$

$$\omega = \sqrt{\frac{GM}{R^3}}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^3}{GM}}}$$

Quer.

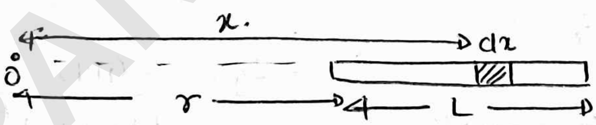


Rod of Mass = M, length = L

① I at O = ?

② V at O = ?

Sol.



$$\frac{dm}{dx} = \frac{M}{L}. \quad \rightarrow dm = \frac{M}{L}.dx$$

$$\int_0^I dI_g = \int -\frac{G(dm)}{x^2}.$$

$$I_g = \int_r^{L+r} -\frac{GM}{L}\cdot\frac{dx}{x^2}$$

$$= \frac{GM}{L}\left[\frac{1}{x}\right]_r^{L+r}$$

$$I_g = \frac{GM}{L}\left[\frac{1}{L+r} - \frac{1}{r}\right]$$

$$I_g = \frac{GM}{L}\left[\frac{r - L - r}{r(L+r)}\right]$$

$$\boxed{I_g = -\frac{GM}{r(L+r)}}$$

*

$$\int dV = -\int \frac{G(dm)}{x}$$

$$V = -\int_{r}^{L+r} \frac{GM}{L} \cdot \frac{dx}{x}$$

$$V = -\frac{GM}{L} \log_e \frac{L+r}{r}$$

$$\boxed{V = \frac{GM}{L} \log_e \frac{r}{L+r}}$$

SPHERICAL MASS DISTRIBUTION

* Hollow :-
(spherical shell)

$$I = -\frac{GM_{inside}}{r^2}$$

$$V = -\int \vec{I} \cdot \vec{dr}$$

① $r < R$

* $I_{in} = 0$

* V_{in}.

$$*\ V = \left[-\int_{\infty}^{R} I \cdot dr\right] + \left[-\int_{R}^{r} \cancel{I_{in}}^{\,0} \cdot dr\right]$$

$$\boxed{V = -\frac{GM}{R}}$$

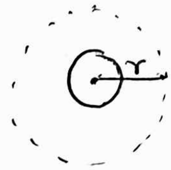
② $r = R$

$$I_s = -\frac{GM}{R^2}$$

$$V_s = -\frac{GM}{R}$$

$$= -\int_{\infty}^{R} I_{out} \cdot dr$$

③ $r > R$

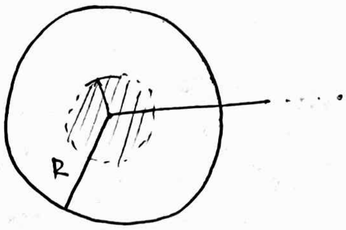


$$I_{out} = -\frac{GM}{r^2}$$

$$V_{out} = -\frac{GM}{r}$$

$$= -\int_{\infty}^{r} I_{out} \cdot dr$$

Solid sphere:-



Outer point

$r > R$

$I = -\frac{GM}{r^2}$

$V = -\frac{GM}{r}$

Surface point

$r = R$

$I = -\frac{GM}{R^2}$

$V = -\frac{GM}{R}$

Inner Point

$r < R$

$M_{in} = \frac{M}{R^3} r^3 = \frac{Mr^3}{R^3}$

$I_{in} = -\frac{G}{r^2}\left[\frac{Mr^3}{R^3}\right]$

$= -\frac{GMr}{R^3}$

☆ V at inner point

$$\Delta V = -\int \vec{I}\cdot\vec{dr}$$

$$= \left[-\int_{\infty}^{R} I_{out}\cdot dr\right] + \left[-\int_{R}^{r} I_{in}\, dr\right]$$

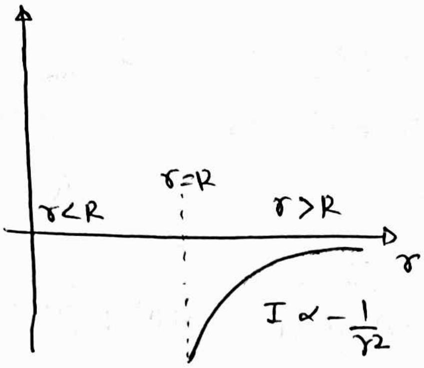
$$= \left(-\frac{GM}{R}\right) + \left(\frac{GM}{R^3}\int_{R}^{r} r\cdot dr\right)$$

$$= -\frac{GM}{R} + \frac{GM}{R^3}\frac{(r^2 - R^2)}{2}$$

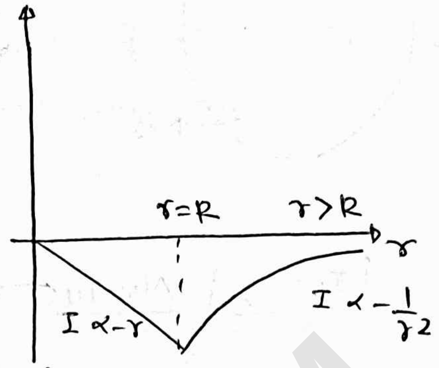
$$\boxed{V_{in} = -\frac{GM}{2R^3}(3R^2 - r^2)}$$

★ Graph for Intensities variat^n :-

* **for Hollow sphere**

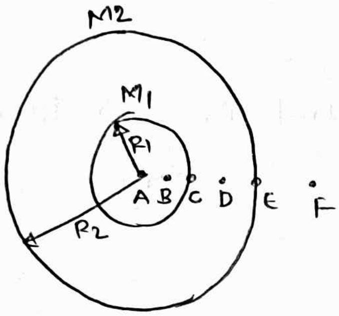


* **for solid sphere**



pyq

* Two concentric shells :-



Intensities

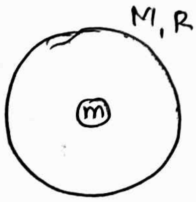
* $I_A = I_B = 0$
* $I_C = -\frac{GM_1}{R_1^2}$
* $I_D = -\frac{GM_1}{r^2}$
* $I_E = -\frac{GM_1}{R_1^2} - \frac{GM_2}{R_2^2}$
* $I_F = -\frac{GM_1}{r^2} - \frac{GM_2}{r^2}$

Potential

* $V_A = V_B = -\frac{GM_1}{R_1} - \frac{GM_2}{R_2}$
* $V_C = -\frac{GM_1}{R_1} - \frac{GM_2}{R_2}$.
* $V_D = -\frac{GM_1}{r} - \frac{GM_2}{R_2}$
* $V_E = -\frac{GM_1}{R_2} - \frac{GM_2}{R_2}$.
* $V_F = -\frac{GM_1}{r} - \frac{GM_2}{r}$

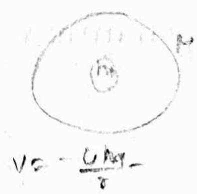
Ques. A particle of mass (m) is kept at the center of a shell of mass (M) + Radius (R), find the potential at a point at distance $\frac{R}{2}$ from center.

Sol.

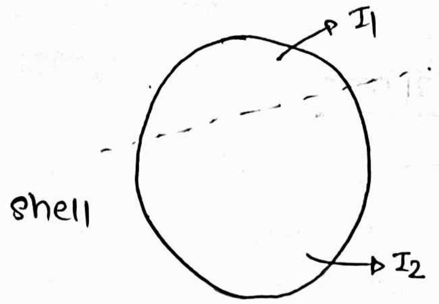


$$V = \left[-\frac{GM}{R}\right] + \left[-\frac{Gm}{R/2}\right]$$

$$\boxed{V = -\frac{G}{R}\left[M + 2m\right]}$$



Ques.



correct relat^n is:-

(A) $I_1 > I_2$

(B) $I_1 < I_2$

(C) $I_1 = I_2$

Sol.

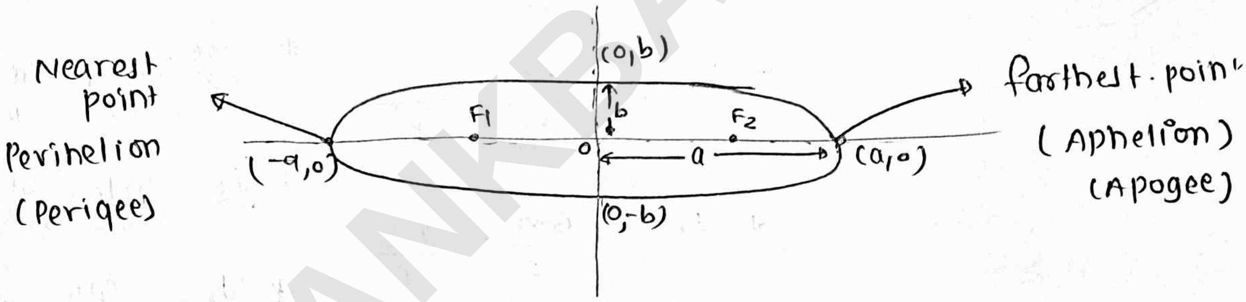
$\boxed{I_1 = I_2}$ Always whether cut equally or unequally

Mistake:- Do not relate it with Mass.

KEPLER'S LAW OF PLANETARY MOTION

* Kepler's 1st law:-

- Aka law of orbits
- Acc. to it All the planets revolve around the sun in elliptical orbits considering sun is located at one of its foci.



$\boxed{PF_1 + PF_2 = const} \Rightarrow$ condit^n for ellipse.

Major Axis $= 2a$

Semimajor axis $= a$

Minor axis $= 2b$

Semiminor axis $= b$.

$F_1 \ (-ae, 0)$

$F_2 \ (ae, 0)$

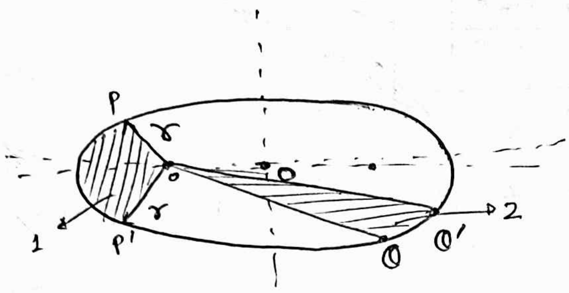
$e =$ eccentricity

$e = \frac{\sqrt{a^2 - b^2}}{a}$

* Kelper 2nd law

* AKa "law of Areas".

* Acc. to this law the radius vector joining the planet to the sun covers (sweeps out) equal areas in equal time intervals



Radius Vector = A vector from sun to planet

$\frac{1}{2}(b)(h) = \Delta$

* Areal velocity = Rate of change in Area $= \frac{dA}{dt} =$ const.

$dA_1 = \frac{1}{2}(r_1)(v_1 dt_1)$

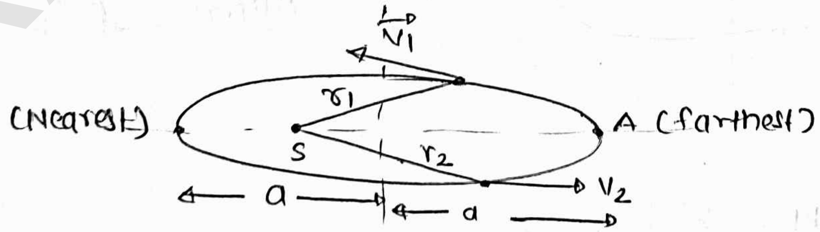
$dA_2 = \frac{1}{2} r_2 (v_2 dt_2)$ If $(dt_1) = (dt_2)$ then $dA_1 = dA_2$

$$\frac{dA_1}{dt_1} = \frac{dA_2}{dt_2}.$$

$$\frac{1}{2} r_1 v_1 = \frac{1}{2} r_2 v_2.$$

$$\boxed{r_1 v_1 = r_2 v_2}$$ $$\boxed{r \propto \frac{1}{v}}$$

* Based on Angular momentum conservatn.



$v_1 r_1 = v_2 r_2$

$v_p r_p = v_a r_a$ $v_p = v_{max}$

$r_p = r_{min} = a - ae = a(1-e)$ $v_a = v_{min}$

$r_a = r_{max} = a + ae = a(1+e)$

$$\boxed{\frac{v_p}{v_a} = \frac{r_a}{r_p}}$$

$$\boxed{\frac{V_p}{V_a} = \frac{V_{max}}{V_{min}} = \frac{r_{max}}{r_{min}} = \frac{1+e}{1-e}}$$

$$V_p = V_{max} = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)}$$

$$V_a = V_{min} = \sqrt{\frac{GM}{a}\left[\frac{1-e}{1+e}\right]}$$

Ques. Determine semimajor $= a$ & semiminor $= b$ in terms of r_{max} & r_{min}

Sol.

$$r_{max} = a + ae \qquad e = \sqrt{\frac{a^2 - b^2}{a}} \qquad b^2 = a^2 - a^2e^2.$$

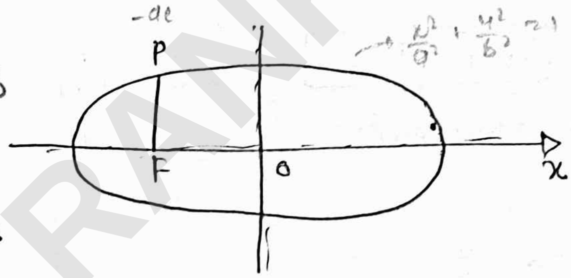
$$r_{min} = a - ae.$$

$$a = \frac{r_{max} + r_{min}}{2} = \text{Avg. Radius of orbit.}$$

$$b^2 = r_{max} \cdot r_{min}$$

$$\boxed{b = \sqrt{r_{max} \cdot r_{min}}}$$

Ques. Aiims AIPMT-1988



length of PF = ?

coordinates of P = ?

PF ⊥ x-axis

Sol.

$$\boxed{PF = \frac{b^2}{a}} \quad \text{Always.}$$

$$\boxed{PF = \frac{2 r_{max} \cdot r_{min}}{r_{max} + r_{min}}}$$

Kepler's 3rd law

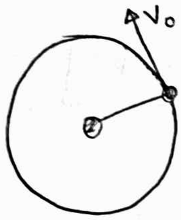
* AKA law of time period

* Acc. to it square of time period of revolutⁿ of planet around the sun is directly proport. to cube of ~~same~~ semi major axis of the elliptical orbit.

i.e $\boxed{T^2 \propto a^3}$

* For circular orbits $\boxed{T^2 \propto R^3}$

Explainatⁿ (circular orbit)



$$\frac{GMm}{r} = \frac{mV_0^2}{r}$$

$$\boxed{V_0 = \sqrt{\frac{GM}{r}}}$$

* $T = \frac{2\pi r}{V_0}$ $\quad T = \frac{2\pi}{\sqrt{GM}} \cdot r\sqrt{r}$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

→ Kepler const.

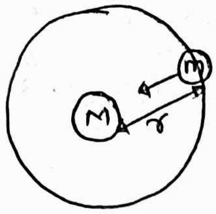
$T = \frac{2\pi}{\omega}$

$\omega^2 = \frac{4\pi^2}{T^2}$ $\quad GM = \omega^2 r^3$

$$\boxed{\omega^2 \propto \frac{1}{r^3}}$$

SATELLITE MOTION :-
(for circular orbits)

if orbit changes All the things described under will changed.



* orbital velocity $\Rightarrow$ $\frac{mv_0^2}{r} = \frac{GMm}{r^2}$

$$\boxed{v_0 = \sqrt{\frac{GM}{r}}}$$

* Period of revolutⁿ $\Rightarrow$ $T = \frac{2\pi r}{v_0}$

$$T = \frac{2\pi}{\sqrt{GM}} r^{3/2}$$

$$\boxed{T^2 = \frac{4\pi^2}{GM} \cdot r^3} \Rightarrow \boxed{T^2 \propto R^3}$$

* Energy of satellite

$$\boxed{T.E = K.E + P.E}$$

$$\boxed{E = K + U}$$

* $K.E = \frac{1}{2} m v_0^2$

$= \frac{GMm}{2r}$

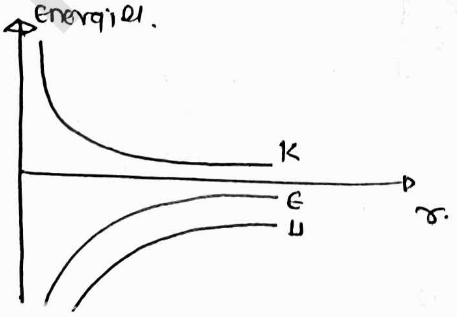
* $P.E = -\frac{GMm}{r}$

Binding energy.

$$\boxed{E = -\frac{GMm}{2r}}$$

* $\boxed{K = -\frac{U}{2}}$ $\boxed{K = -E}$ & $\boxed{U = 2E}$ $\boxed{E = -ve}$
↓
bounded state.

Graphs



$K \propto \frac{1}{r}$

$U = -\frac{1}{r}$

$E = -\frac{1}{r}$.

BINDING ENERGY

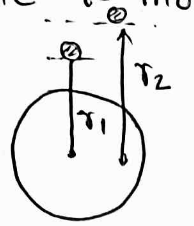
→ Numerically it is equal to the -ve of Total energy

$$\boxed{B.E = -TE}$$

→ it is energy required for a satellite to separate it from gravitational attraction of Planet.

→

Ques. A particle of mass is kept at dist r_1 from center of Planet. Find work Done to move it from r_1 to r_2?



Sol.

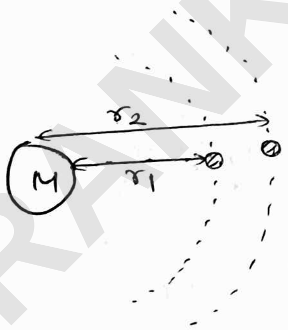
$$W.D = \Delta U = U_F - U_i$$

$$\Rightarrow -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right)$$

$$\boxed{\Delta U \Rightarrow GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$$

Ques. Find Work done to Ies the radius of stallelite's orbit to r_1 to r_2?

Sol.



$$W.D = E_F - E_I$$

$$\Rightarrow -\frac{GMm}{2r_2} - \left(-\frac{GMm}{2r_1}\right)$$

$$\boxed{\Delta E = \frac{GMm}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$$

Ques. A staellite of a planet is orbiting in orbit of radius ~~small~~ 'r' & with Angular Momentum L. If we Ies radius of orbit to 16r then Angular momentum becomes

→ Angular Momentum is constant in same orbit.

Sol →

$$L = mv_0 r$$

$$\Rightarrow m\sqrt{\frac{GM}{r}}\cdot r$$

$$\boxed{L \propto \sqrt{r}}$$

$$\frac{L_2}{L_1} = \sqrt{\frac{r_2}{r_1}}$$

$$\boxed{L' = 4L}$$

TYPES OF SATELLITE

(A) POLAR SATELLITE

→ Int in Plane of Polar axis

→ $r \approx R$

$$V_o = \sqrt{\frac{GM}{R}} = \frac{V_e}{\sqrt{2}} = 8 \text{ km/s.}$$

* V_o को $\sqrt{2}$ times कर दे तो
* Additional 3.2 km/s spd.
* K.E को Double करे
* K.E को 100% ↑es
* orbital spd को 41.4% ↑es कर दे तो

satellite escape कर जायेगा।

(B) GEO-STATIONARY SATELLITES :- (GSS)

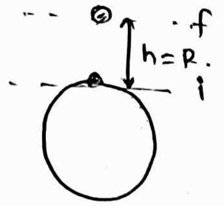
$T = 24 \text{ hr.}$

$$T^2 = \frac{4\pi^2}{GM} r^3.$$

$\boxed{r \approx 7R}$ At a height of 6R from earth surface.

Ques. Find work done to displace a point mass m_0 from surface of earth (M, R) to a point at a Height equal to radius of earth from surface.

sol.



$$V_i = -\frac{GM}{R}$$

$$V_f = -\frac{GM}{R+h} = -\frac{GM}{2R}$$

$$W_{agent} = M_0(V_f - V_i)$$

$$= m_0\left(-\frac{GM}{2R} + \frac{GM}{R}\right)$$

$$\boxed{W_a = \frac{GMm}{2R}} =$$

$$W_a = \frac{gR^2 m}{2R} = \frac{1}{2} mgR.$$

Ques.

attracts each other.

$v = ?$

sol.

$$\cancel{K_{\infty}}^{0} + \cancel{U_{\infty}}^{0} = K_2 + U_2$$

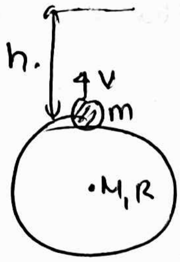
$$2\left(\frac{1}{2}mv^2\right) - \frac{GM^2}{r} = 0$$

$$mv^2 = \frac{GM^2}{r}$$

$$\boxed{v = \sqrt{\frac{GM}{r}}}$$

Que. An object is projected with $\left(\frac{V_e}{\sqrt{2}}\right)$ then find Hmax.

Sol.



$$\cancel{K_1}^{0} + U_1 = \cancel{K_2}^{0} + U_2$$

$$\frac{1}{2}mv^2 + \left(-\frac{GmM}{R}\right) = \frac{-GMm}{(R+h)}$$

$$\boxed{h = R}$$

Que. if $F_g \propto \frac{1}{r^n}$ then find dependency of orbital spd. (V_o) & time period (T) of a satellite on r.

Sol.

$$F_g = \frac{K}{r^n} \qquad F_c = \frac{mV_o^2}{r}$$

for motn $\quad F_c = F_g$.

$$\frac{mV_o^2}{r} = \frac{K}{r^n}$$

$$V_o^2 = \frac{Kr}{r^n}$$

$$V_o^2 \propto r^{1-n}$$

$$\boxed{V_o \propto r^{\frac{1-n}{2}}}$$

$$T = \frac{2\pi r}{V_o} \qquad T \propto \frac{r}{V_o}$$

$$T \propto \frac{r^1}{r^{\frac{1-n}{2}}}$$

$$T \propto r^{1-\left[\frac{1-n}{2}\right]}$$

$$\boxed{T \propto r^{\frac{1+n}{2}}}$$