* SOLIDS *

* **elasticity** :- it is property of Material by which it can oppose the deformatn
  → depends only on the Nature of Material.

* **Deforming Force**:- It is an external force which generates deformatn in a material.

* **Restoring force** :- Numerically equal to deforming force
  → it is an internal Force by the material which opposes the deformatn.
  → it will not act until ~~is~~ deforming force is not removed

HOOK's LAW

⇒ Acc. to this the deformatn ~~due~~ produced is proportional to external deforming force

$\boxed{F \propto x}$ $x =$ deformatn.

* when deforming force is removed within elastic limit then restoring force (due to nature of Material) starts to act.

$\boxed{F_R \propto -x}$ −ve shows the opp. dirn as compared to deformatn

Elastic limit

* Numerically when $\Delta l \le 1\%$ of length, then Hook's law will be followed.

* Elastic Materials

→ Those materials which can regain their ~~sph~~ shape after removal of deforming force.

→ Those materials which are hard to deform.

* Most elastic ⇒ Quartz, steel

* Plastic Materials ⇒

→ Those materials which can't regain its shape & conf. after removal of deforming force.

16/12/19

L-2

TL-74

STRESS

* It is defined as restoring force per unit area.

i.e

$$\boxed{\text{Stress} = \frac{\text{Restoring Force}}{\text{Area of cross sec.}}}$$

STRESS

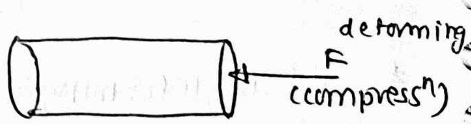
① Longitudinal stress
↓
Along the length

F = Normal to C.S.

* length ↑es

* Tensile longi. stress.

* intermolecular force = attractⁿ

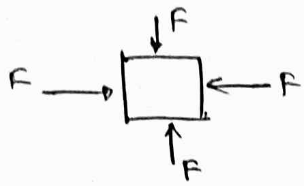


* length decreases

* compression longitudinal stress.

* inter molecular force = repulsion.

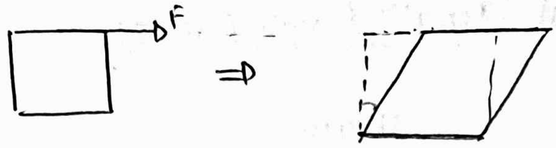
② volume stress
↓
like "pressure"



size → ~~un~~change
shape → unchange
Vol. → changes

③ <u>Shear stress</u>

→ Deforming force is tangential to c.s



$$\boxed{\text{Shear stress} = \frac{F_{\text{tangential}}}{A}}$$

④ <u>Breaking stress</u>

→ Maximum stress developed to break the material.

* <u>Depends on</u>

① Nature of the material.

② Temperature. (temp. directly relates with intermolecular forces)

⑤ <u>Thermal stress</u>

↓

longitudinal stress.

$$\boxed{Y = \frac{\text{longi. stress}}{\text{longi. strain}}}$$

⇒ <u>Thermal stress</u> $= Y\left(\frac{\Delta L}{L}\right)$

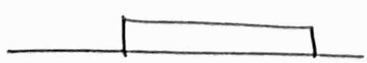
$= Y(\alpha \Delta T)$ $\left(\because \boxed{L = L_0(1+\alpha\Delta T)}\right)$

As temp. ↑, length ↑es.

*

$$\boxed{\text{Thermal stress} = \frac{\text{Force due to clamp}}{\text{Area of c.s}}}$$

ex:-



elastic metal rod is kept on horizontal frictionless surface.

* if temp. ↑, length ↑

$\frac{\Delta L}{L} = \alpha\Delta T$

* $\boxed{\text{Thermal stress} = 0}$

coz there is force to opp. its elongation

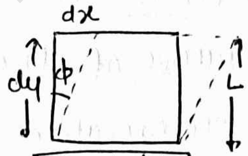
STRAIN

$$\boxed{\text{strain} = \frac{\text{change in length / vol / shape}}{\text{original length / vol / shape}}}$$

unitless
&
Dimensionless.

strain

longitudinal strain	vol. strain	shear strain OR angle of shear
$= \frac{\text{change in length}}{\text{original length}}$	$= \frac{\text{change in vol.}}{\text{original vol.}}$	



$\tan\phi = \phi = \frac{dx}{dy}$.

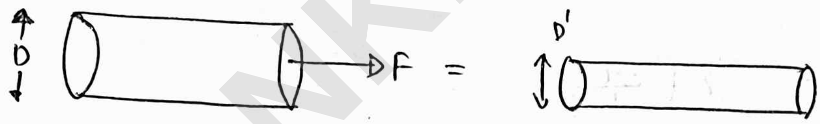
~~$\phi = dx/dy$~~

$\phi = \frac{(dx)}{L}$

* vol → same.
shape → change.

* longitudinal strain :-

$\frac{\Delta L}{L} = +ve$ (Generally)



* lateral strain ⇒ in diameter $= \frac{\Delta D}{D} = -ve$ (Generally)

* Poisson's Ratio = $\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{-(\Delta D/D)}{(\Delta l/l)}$

* Theoretical value → $-1 \le \sigma \le 0.5$
* practical value → $0.2 \le \sigma \le 0.4$

* cylindrical wire

$V = \frac{\pi D^2 l}{4}$

$\frac{\Delta V}{V} = 2\frac{\Delta D}{D} + \frac{\Delta l}{l}$

$\sigma = -\frac{\Delta D/D}{\Delta l/l} = \frac{1}{2} = 0.5.$

↓

σ_{max}.

V = const.

$\frac{\Delta V}{V} = 0.$

$\frac{\Delta l}{l} = -2\frac{\Delta D}{D}$

★ Modulus of Elasticity (E):-

Hooke's law :-

it was applicable for elongat^n only when deforming is applied, than elongat^n or extension is produced such that $F \propto x$.

$\boxed{F \propto x}$ → x = expans^n ; → x = compress^n ⟹ x = deformat^n.

⇓

$F_R \propto -x$ ⟹ Restoring force. Acts opp. to deformat^n.

★ Young Modified this law as ⟹ $\boxed{\text{stress} \propto \text{strain}}$

⟹ Applicable for every deformat^n (elongat^n, compression, twisting etc.).

* Stress = E (strain)

$$\boxed{\text{Modulus of Elasticity } (E) = \frac{\text{Stress}}{\text{Strain}}}$$

Depends on ① Nature of Material
② Temperature.

E

- Y — Young Modulus (for solids)
- Bulk — Bulk Modulus. (For solids, liq., gas)
- η. — Modulus of rigidity.

$$\boxed{Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A(\Delta L)}}$$

$$\boxed{K = \frac{\Delta P}{-(\Delta V/V)}}$$

$$\boxed{\eta = \frac{F_{tang t.}}{A\phi}}$$

unit + Dimensions of

$$\boxed{E(Y, K, \eta) = \text{stress}}$$

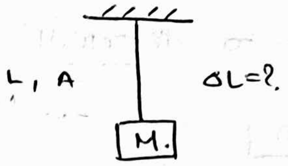
YOUNG'S MODULUS

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

$$\boxed{\Delta L = \frac{FL}{AY}}$$

F = Tension.

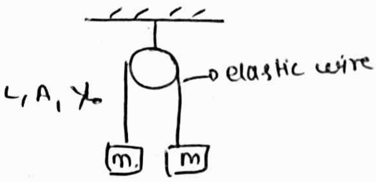
Que.



Sol.

$$\boxed{\Delta L = \frac{MgL}{AY}}$$

Que.



Sol.

$$\Delta L = \frac{FL}{AY}$$

$$\Delta L = \Delta L_1 + \Delta L_2$$

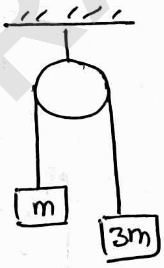
$$= \frac{Mg(L/2)}{AY\cancel{2}} + \frac{Mg(L/2)}{AY}$$

$$\boxed{\Delta L = \frac{MgL}{AY}}$$

$$T = \frac{2m_1m_2g}{m_1+m_2}$$

Que.

$\Delta L = ?$



$3mg - T = 3ma$

$T - mg = ma$

$2mg = 4ma$

$a = g/2$

Sol.

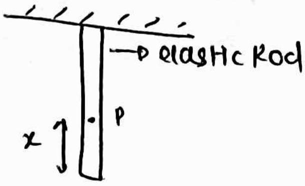
$$T = \frac{2(3m)(\cancel{m})g}{4\cancel{m}}$$

$$= \frac{3}{2}mg$$

$$\Delta L = \Delta L_1 + \Delta L_2$$

$$= 2\times\frac{\frac{3}{\cancel{2}}mg\times\frac{L}{2}}{AY} = \boxed{\Delta L = \frac{3mgL}{2AY}}$$

Que.



M, L, A, Y

'P' पर stress = ?

$\Delta L = ?$

↳ extensⁿ due to its own weight.

Sol. ① $\text{stress} = \frac{F}{A}$

$$= \frac{\left(\frac{M}{L}x\right)g}{A} = \frac{Mxg}{AL}$$

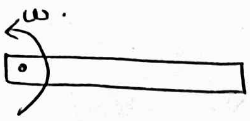
$\boxed{\frac{Mxg}{LA}}$

$\Delta l = \frac{Mxg}{L} \times \frac{L}{AY} \int_0^x \frac{Mxg}{AY}$

$\frac{F}{A}$

② $\boxed{\Delta L = \frac{Mg(L/2)}{2AY}}$ 1/2 for COM.

Que.



$\Delta L = ?$

Sol.

$$\Delta L = \frac{FL}{AY}$$

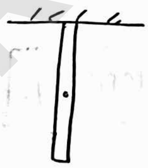
$F = \int_0^L m\omega^2 x\,dx$

$$\Delta L = \frac{M\omega^2 L}{2} \times \frac{L}{AY}$$

$$\boxed{\Delta L = \frac{M\omega^2 L^2}{2AY}}$$

Que. An elastic rod is suspended from ceiling its density ρ, L, A, Y. find extensⁿ produced due to its own weight. ?

Sol.



$$\Delta L = \frac{MgL}{2AY}$$

$$\Delta L = \frac{\rho V g L}{2AY}$$

$$\Delta L = \frac{\rho(AL)gL}{2AY}$$

$$\boxed{\Delta L = \frac{\rho g L^2}{2Y}}$$

does not depend on Area of cross sectⁿ.

* Potential energy stored in a streched wire.

→ $F = Kx$

→ $F = \overbrace{\left(\frac{AY}{L}\right)}^{K}\overbrace{(\Delta L)}^{x}$

$U = \frac{1}{2}Kx^2$

$$\boxed{U = \frac{1}{2}\left(\frac{AY}{L}\right)(\Delta L)^2}$$

* Energy density:-

$$\boxed{u = \frac{U}{V}} \qquad V = AL$$

$$\boxed{u = \frac{1}{2}Y\left[\frac{\Delta L}{L}\right]^2}$$

$$\boxed{u = \frac{1}{2}Y(\text{strain})^2}$$

$Y = \frac{\text{stress}}{\text{strain}}$

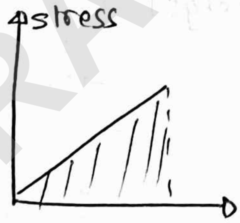
$$\boxed{u = \frac{1}{2}\frac{(\text{stress})^2}{Y}}$$

$$\boxed{u = \frac{1}{2}(\text{stress})(\text{strain})}$$

Graph

under Hook's law condit^n :-

when (stress ∝ strain)

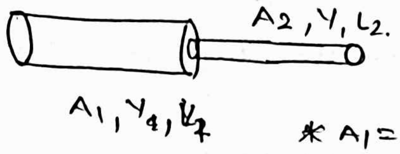


* $\boxed{\text{slope} = Y}$

* $\boxed{\text{Area} = \text{energy density.}}$

combinatn of Elastic Rods (same as spring):-

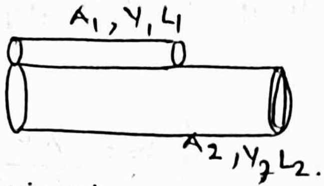
SERIES



* $A_1 = A_2$

$$\boxed{\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}}$$

* **Parallel**



$L_1 = L_2$.

$$\boxed{K = K_1 + K_2}$$

* **Equivalent Young Modulus.**

$$\frac{L_1 + L_2}{A(Y)} = \frac{L_1}{AY_1} + \frac{L_2}{AY_2}.$$

$$Y = \frac{L_1 + L_2}{\frac{L_1}{Y_1} + \frac{L_2}{Y_2}} = \frac{\Sigma L}{\Sigma\left(\frac{L}{Y}\right)}$$

if $L_1 = L_2$.

$$\boxed{Y = \frac{2Y_1 Y_2}{Y_1 + Y_2}}$$

* **Equivalent Young Modulus**

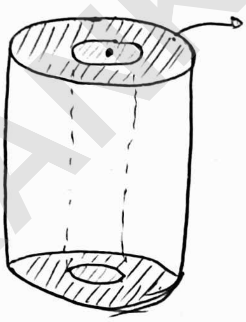
$$\frac{(A_1 + A_2)Y}{L} = \frac{A_1 Y_1}{L} + \frac{A_2 Y_2}{L}$$

$$\boxed{Y = \frac{A_1 Y_1 + A_2 Y_2}{A_1 + A_2}}$$

if $A_1 = A_2$.

$$\boxed{Y = \frac{Y_1 + Y_2}{2}}$$

Ques. Equivalent 'Y' for a composite Rod :-



outer radius $= r_2$
inner radius $= r_1$

Hint:- if on applying force if ΔL is same then.
it is a $\parallel^{le}$ combinatn.

$K = K_1 + K_2$ $\qquad$ $Y = \frac{\Sigma(AY)}{\Sigma(A)}$

outer $= Y_2$.
inner $= Y_1$

$Y_{comb} = ?$

$$Y = \frac{\pi r_1^2 (Y_1) + (\pi r_2^2 - \pi r_1^2) Y_2}{\pi r_2^2}.$$

$$\boxed{Y = \frac{Y_1 r_1^2 + Y_2(r_2^2 - r_1^2)}{r_2^2}}$$

Ques. find value of

① isothermal Bulk Modulus. OR isothermal elasticity of gas.

② Adiabatic Bulk Modulus or Adiabatic elasticity of Gas.

Sol.

$$\boxed{B = \frac{\Delta P}{-(\Delta V/V)}}$$

$$\boxed{\text{compressibility } (c) = \frac{1}{B}}$$

isothermal

$PV = *$

$P(\Delta V) + V(\Delta P) = 0.$

$P\Delta V = - V\Delta P$

$$B = \frac{\Delta P}{-\left(\frac{\Delta V}{V}\right)} = P$$

$$\boxed{E_{isothermal} = B_{isoth.} = P}$$

Newton

Adiabatic.

$PV^{\gamma} = *$

$P[\gamma V^{\gamma-1}.\Delta V] + V^{\gamma}[\Delta P] = 0$

$\gamma P\left[\frac{\Delta V}{V}\right] + \Delta P = 0$

$$B = \frac{\Delta P}{-[\Delta V/V]} = \gamma P$$

$$\boxed{B_{adiabatic} = \gamma P}$$

Laplace correctn → sound wave.

Relation Among Y, K, η, σ :-

$\boxed{\cancel{Y = 2K(1-\sigma)}}$

* $\boxed{Y = 3K(1-2\sigma)}$

* $\boxed{Y = 2\eta(1+\sigma)}$

* $\boxed{\frac{3K}{2\eta} = \frac{1+\sigma}{1-2\sigma}}$

* $\boxed{\frac{Y}{3K} + \frac{Y}{\eta} = 3}$

$\Downarrow$

$$\boxed{\frac{9}{Y} = \frac{1}{K} + \frac{1}{\eta}}$$

Que. when a stress of $10^8\ N/m^2$ is applied on an elastic wire, then change in its length is 10^{-3} times of its original length find Young Modulus

sol

$$\frac{F}{A} = 10^8$$

$$\frac{\Delta L}{L} = 10^{-3} \qquad Y = \frac{F}{A}\frac{L}{\Delta L}$$

$\frac{F}{A} = 10^8$

$\frac{\Delta L}{L} = 10^{-3}$

$Y = \frac{F}{A}\frac{\Delta L}{L}$

Que. $Y = 5\times10^{10}\ N/m^2$

$K = 10^{12}\ N/m^2$

$\sigma = ?$ (Poison's ratio).

Sol.

$$Y = 3K(1-2\sigma)$$

$$5\times10^{10} = 3\times10^{12}\,(1-2\sigma)$$

$$\frac{5}{3}\times10^{-2} = 1-2\sigma$$

$$2\sigma = 1 - \frac{5\times10^{-2}}{3}$$

$$2\sigma = \frac{300-5}{300}$$

$$2\sigma = \underline{295}$$

Que. $Y = 2.4$ times of its Modulus of rigidity (η). find poisson's ratio.

sol. $\boxed{Y = 2.4\eta}$

$$Y = 2\eta(1+\sigma).$$

$$2.4\cancel{\eta} = 2\cancel{\eta}(1+\sigma)$$

$$1.2 = 1+\sigma$$

$$\boxed{\sigma = 0.2}$$

Que. Length of wire is L_1 when $F_1 = 4N$ is applied
length of wire is L_2 when $F_2 = 5N$ is applied.
find L_3 if $F_3 = 9N$ is applied.

Sol.

$F = Kx$

assumed that initial length is L_0.

$F_1 = K(L_1 - L_0)$

$F_2 = K(L_2 - L_0)$.

$F_3 = K(L_3 - L_0)$.

$Y = \frac{F_1 L}{A \Delta L}$

$\Rightarrow \frac{4 L_1}{A \Delta L}$

$\frac{4}{K} = L_1 - L_0$

$\frac{5}{K} = L_2 - L_0$.

$\frac{1}{K} = L_2 - L_1$

$\frac{4}{5} = \frac{L_1 - L_0}{L_2 - L_0}$.

$4L_2 - 4L_0 = 5L_1 - 5L_0$.

$L_0 = 5L_1 - 4L_2$.

Ans. $\boxed{L_3 = L_0 + \frac{F_3}{K}}$

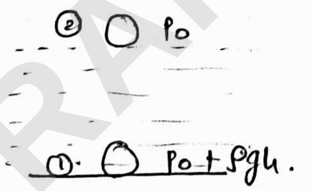
$L_3 = 5L_1 - 4L_2 + \underline{9}(L_2 - L_1)$

$\Rightarrow 5L_1 - 4L_2 + 9L_2 - 9L_1$

$\boxed{L_3 = 5L_2 - 4L_1}$

Que. An air bubble rises up in a lake of depth 'h' when it reaches to the surface find fractional change in its vol.?

Sol.



$\Delta P = -\rho g h$.

$\Downarrow$

vol. will inc.

$\frac{\Delta V}{V}$ = fractional change. = +ve

$B = \frac{\Delta P}{-(\Delta V / V)}$

$\frac{\Delta V}{V} = \frac{\Delta P}{-B} = \frac{-\rho g h}{-B}$.

$\boxed{\frac{\Delta V}{V} = \frac{\rho g h}{B}}$ or $\boxed{\frac{\Delta V}{V} = C(\rho g h)}$

Que. When a sphere is taken to bottom of lake its vol. is $\downarrow$ed by 0.01%.
depth of lake = 1km.
Find B.

sol.

$$\frac{\Delta V}{V} = \frac{\rho g h}{B}$$

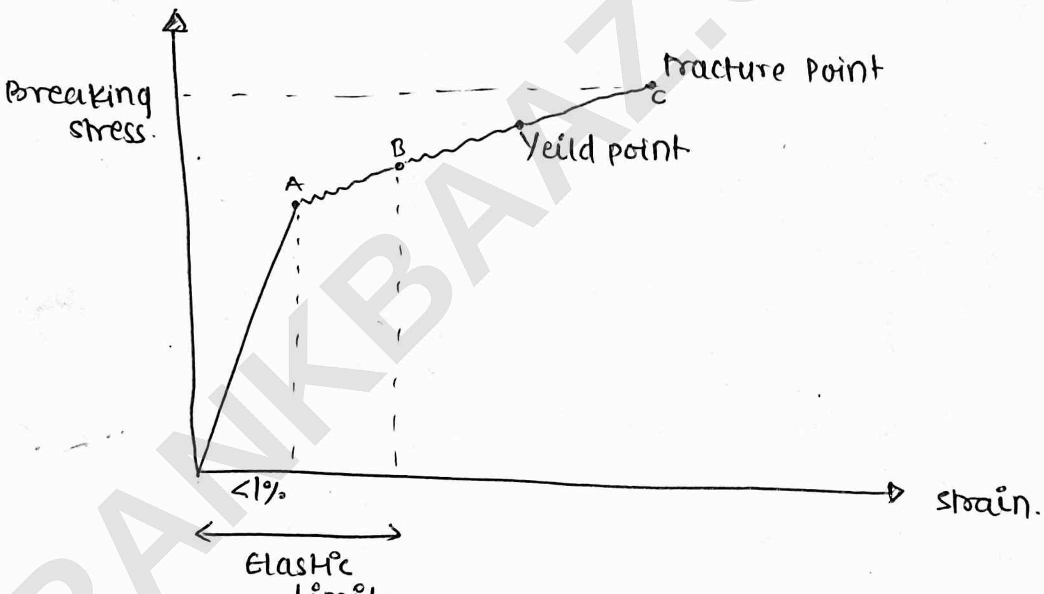
$$\frac{\Delta V}{V} = \frac{0.01}{100} = 10^{-4}$$

$$B = \frac{10^{3} \times 10 \times 10^{3}}{10^{-4}}$$

$$B = 10^{+7} \times 10^{4}$$

$$\boxed{B = 10^{11}\ N/m^2}$$

STRAIN - STRESS GRAPH



OA = $\underline{\text{Obeys}}$ Hook's law.
OB = elastic

YF → small = Brittle.
YF → Big = Tencile Nature.