23/12/19

FLUID - STATICS

* study of Fluid at rest
* Intermolecular forces in liquids are weaker as compared to solids.
* liquids are incompressible i.e density = *
* They take shape of container. but not vol.
* They have free surface of their own.
* $\boxed{\text{Gases}}$ are compressible
* They take shape of container as well as volume.

* Density :-
  - Volume Mass density
  * Mass per unit volume.

$$\rho = \frac{M}{V}$$

* unit :- SI = Kg/m^2
  C.G.S = g/cm^3 or g/cc

* water :- 4°c
  $1 g/cm^3$ or $1 g/ml$ or $1000\ Kg/m^3$.

② Mercury $13.6\ g/cm^3$
or
$13.6 \times 10^3\ Kg/m^3$

* density of Material / substance = const.
* density of body ⇒ depends on body "shape".

Solid Bodies

* $M_{Body} = M_{subs.}$
* $V_{Body} = V_{subs.}$
* $\rho_{body} = \rho_{subs.}$

Hollow bodies

* $M_{body} = M_{subs.}$
* $V_{body} > V_{subs.}$
* $\rho_{body} < \rho_{subs.}$

$$\boxed{\rho_{mix} = \frac{M_1+M_2}{V_1+V_2}}$$

M & ρ given

$$\rho_{mix} = \frac{M_1+M_2}{\frac{M_1}{\rho_1}+\frac{M_2}{\rho_2}}$$

$$\rho_{mix} = \frac{(\Sigma M)}{[\Sigma(M/\rho)]}$$

→ if equal masses are mixed

$M_1 = M_2$.

$$\boxed{\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1+\rho_2}}$$

V & ρ given.

$$\rho_{mix} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1+V_2}.$$

$$\rho_{mix} = \frac{(\Sigma \rho V)}{\Sigma(V)}$$

→ if equal vol. ~~masses~~ are mixed

$V_1 = V_2$.

$$\boxed{\rho_{mix} = \frac{\rho_1+\rho_2}{2}}$$

Ques When two substances of density ρ_1 & ρ_2 are mixed ($\rho_1 > \rho_2$) then density of mixture is 4 kg/m³ if equal vol. are mixed & density of mix = 3 kg/m³ if equal mass are mixed?

Sol.

$$3 = \frac{2\rho_1\rho_2}{\rho_1+\rho_2}$$

$$2\rho_1\rho_2 = 24$$

$$\boxed{\rho_1\rho_2 = 12}$$

$$4 = \frac{\rho_1+\rho_2}{2}$$

$$\boxed{\rho_1+\rho_2 = 8.}$$

$\rho_1 = 6$

$\rho_2 = 2.$

$\rho_1 = 8-\rho_2$

$(8-\rho_2)\rho_2 = 12$

$8\rho_2 - \rho_2^2 = 12$

$x^2 - 8x + 12 = 0$

$64 - 4(1)(12)$

$\frac{8 \pm 4}{2} = 6$

Ques. $\rho_1 = 4\ kg/m^3$ $\quad \rho_2 = 2\ kg/m^3$

$\frac{M_1}{M_2} = \frac{3}{1}$

$\rho_{mix} = ?$

Sol.

$$\rho_{mix} = \frac{M_1+M_2}{\frac{M_1}{\rho_1}+\frac{M_2}{\rho_2}} = \frac{3+1}{\frac{3}{4}+\frac{1}{2}} = \frac{4}{\frac{5}{4}} = \frac{4}{1}\times\frac{4}{5}$$

$$\Rightarrow \frac{16}{5}\ kg/m^3.$$

Weight Density

* $\omega = \frac{\text{weight}}{\text{volume}} = \frac{Mg}{V}$

* $\boxed{\omega = \rho g}$

* unit:- N/m^3

Relative Density

* $\boxed{\rho_R = \frac{\rho \text{ of subs. / body}}{\rho \text{ of pure water at } 4^\circ C}}$

$\boxed{\rho_R = \frac{\rho}{\rho_w}}$

* unitless, dimensionless

Specific Gravity

* S.G = Ratio of weight densities

$= \frac{W_{body}}{W_{water}}$

$\boxed{S.G = \frac{\rho}{\rho_w}}$

Numerically equal to Relative density.

PRESSURE :-

- Defined as Normal force per unit surface Area

$P = \frac{dF}{dA}$ or $\boxed{P = \frac{F}{A}}$ unit :- N/m^2

$dyne/cm^2$

* units :- $1 N/m^2 = 1 Pa = 1$ Pascal

(2) $\boxed{\begin{array}{l} 1 \text{ atm} = 1 \text{ bar} = 760 \text{ torr} = 760 \text{ mm of Hg} = 76 \text{ cm of Hg} \\ 1 \text{ atm} = 1 \times 10^5 N/m^2 \end{array}}$

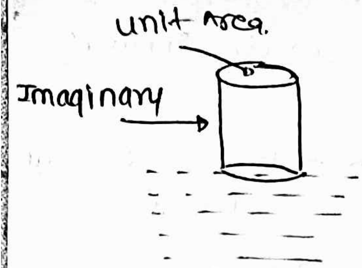
TYPES OF PRESSURE

$\boxed{P_{absolute} = P_{atm} + P_{gauge}}$

* Absolute pressure :- • Net pressure
  • sum of Atm. + Gauge pressure.

* Atmospheric Pressure :-

→ it is the force exerted by air on a unit surface area of sea level.



$F = Mg.$

$F = \rho v g$

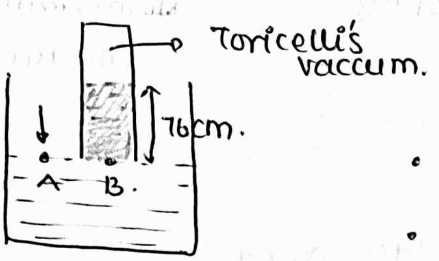
$F = \rho A L g.$

$P = \frac{F}{A}$

$\boxed{P = \rho g h}$

#

Take a Glass tube of uniform cross sect^n + length 1m which is filled with mercury, inverted into a mercury tray.



- $P_A = P_{atm} = P_B$
- $P_B = \rho g h.$ → $\rho = 13.6 g/cm^3$, $h = 76$ cm, $g = 980$ cm/s^2 } 1 Atm = 1.01×10^5 N/m

* "Barometer" is used to measure atmospheric pressure.

GAUGE PRESSURE

It is the excess pressure

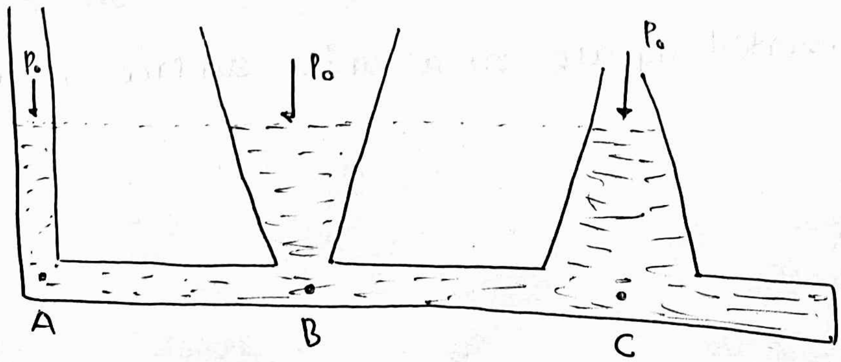
* G.P = Absolute − Atmospheric

* Instrument :- Manometer

* $\boxed{P_{gauge} = \rho g h}$

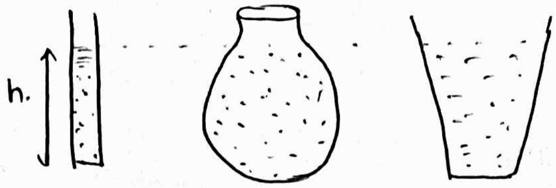
HYDROSTATIC PARADOX :-

- Acc. to it pressure is proportional to height of liquid column.
- It is independent of size + shape of container
- & independent of base area of container.



$$P_A = P_B = P_C = P_0 + \rho g h.$$

*



same pressure.

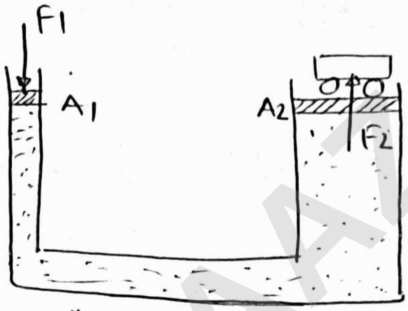
PASCAL'S LAW

* pressure in a fluid at rest is same at all points if gravity is ignored

* if pressure in an enclosed fluid is changed at a particular point then change is transmitted to every point of fluid and to walls of container without being losed in magnitude.

* Pressure Applied

$$= \frac{F_1}{A_1}$$



"Hydroulic lift"

* Pressure transmitted

$$= \frac{F_2}{A_2}.$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\boxed{F_1 = \left(\frac{A_1}{A_2}\right) F_2}$$

Ques.

$A_1 = 1 cm^2$

$A_2 = 10 m^2$

$M_2 = 1000 \, kg$

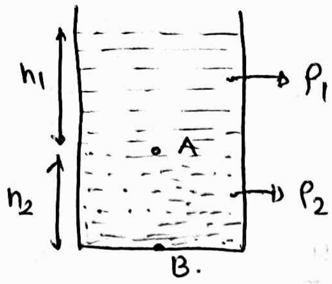
$F_1 = ?$

Sol.

$$F_1 = \frac{10^{-4}}{10} \times 10^3 \times 10$$

$$\boxed{F_1 = 0.1 N}$$

MANOMETERIC EQUATIONS

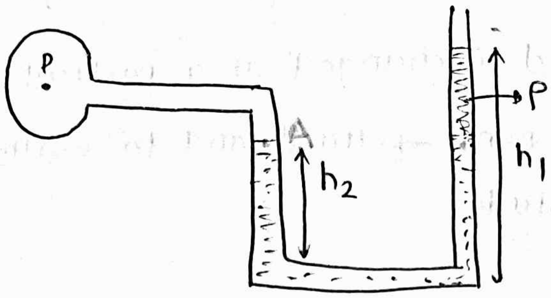


★ $\rho_2 > \rho_1$

✱ $P_A = P_0 + \rho_1 g h_1$

$P_B = P_0 + \rho_1 g h_1 + \rho_2 g h_2.$

Que.



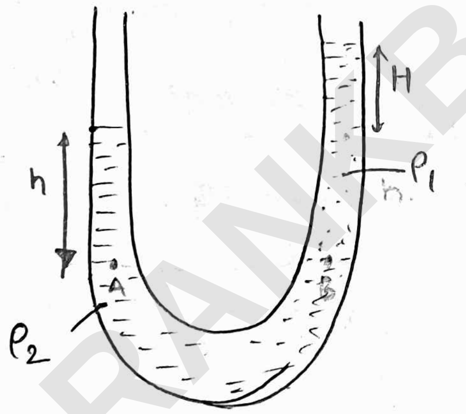
Pressure at P = ?

$P_A = P_0 + \rho g(h_1 - h_2)$

sol.

$$P_A = P_0 + \rho g(h_1 - h_2)$$

Que.
Neet 2017



find H in terms of ρ_1, ρ_2, h.

sol.

$P_A = P_B.$

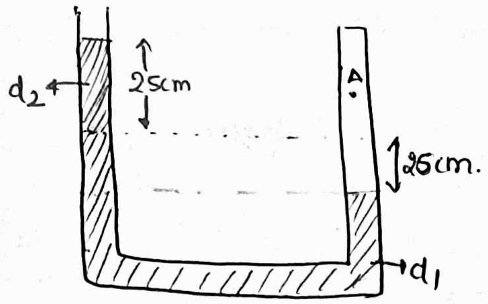
$\Rightarrow \rho_2 g h = \rho_1 g (H + h)$

$\Rightarrow \rho_2 h = \rho_1 H + \rho_1 h$

$\Rightarrow \rho_1 H = \rho_2 h - \rho_1 h$

$\Rightarrow \boxed{H = \left(\frac{\rho_2 - \rho_1}{\rho_1}\right) h}$

Ques.



* Pressure At A ?

$d_2 = 2d_1$

$d_1 = 13.6\ g/cm^3$

Sol.

$P_A = P_B = P_C$

$d_2 g\, 25 + d_1 g\, 26$

$2d_1 g\, 25 + d_1 g\, 26$

$P_c = P_0 + d_2 g (25cm) + d_1 g (26cm)$

$= P_0 + d_1 g (76cm)$

$\boxed{P_c = 2P_0}$

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BUOYANT FORCE

* When a body is submerged partially or completely in a fluid then it experiences an upward force due to the fluid surrounding it.

* Force $\boxed{F_B = V_{in} d_L g}$

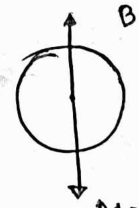
this Phenomenon is c/a Buoyancy & Force is c/a Buoyant force.

* ARCHIMEDE'S PRINCIPLE

→ when a body is submerged partially or completely in a fluid then there is some decrease in its weight is noticed.

→ This decrease in weight is equal to the weight of fluid displaced by the body.

* weight of body = $\boxed{Mg = V_B d_B g}$



$B = F_B =$ Thrust $= V_{in} d_L g$

$Mg = V_B d_B g$

* Thrust ⇒ $\boxed{B = V_{in} d_L g}$

* V_{in} = vol. of body inside liq.

$\boxed{V_{in} \leq V_B}$

* Thrust depends on density of liquid.

APPLICATION:-

1. $W_{air} = W_a$

$W_{liq} = W_L$

$= W_{app.}$

$\Rightarrow \quad W_L = W_a - B$

$$\boxed{B = W_a - W_L}$$

2. Relative Density of an object

$$R.D = \frac{d_B}{d_W} = \frac{d_B V_B g}{d_W V_B g}$$

$$= \frac{\text{weight of the body}}{\text{weight of equal vol. of water}} = \frac{W_a}{B}$$

$$\boxed{R.D = \frac{W_a}{W_a - W_W}}$$

3. R.D of a liquid

$$R.D = \frac{d_L}{d_W} = \frac{d_L/d_B}{d_W/d_B} = \frac{\frac{W_a - W_L}{W_a}}{\frac{W_a - W_W}{W_a}}$$

$$\boxed{\text{R.D of liquid} = \frac{W_a - W_L}{W_a - W_W}}$$

FLOTATION

① $d_B > d_L$ sink ($V_B = V_{in}$).

* $W_{app} = W - \text{Thrust} = V_B d_B g - V_{in} d_L g$

$$= V_B g(d_B - d_L) = W\left[1 - \frac{d_L}{d_B}\right]$$

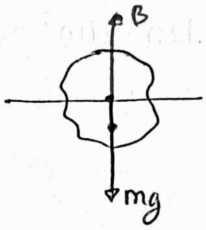
② $d_B = d_L$

↳ float, fully submerged.

* $\boxed{W_{app} = 0}$

③ $d_B < d_L$

↳ float, partially submerged



$B = Mg$

Weight of equal volume (displaced) of water	=	Weight of the whole body.

$$\boxed{V_B d_B g = V_{in} d_L g}$$

$$\boxed{M_B = V_B d_B = V_{in} d_L}$$

Que. A Block of wood is submerged 80% in water & floats. Find density of wood.

Sol.

$V_B d_B = V_{in} d_L$ $V_{in} = \frac{80}{100} V_B$.

$V_B d_B = \frac{80}{100} V_B \times 1000$

$$\boxed{d_B = 800 \ kg/m^3}$$

Que1. A cube of mass M floats in water 75% submerged, when a coin of mass m is placed on it, it floats fully submerged find $\frac{M}{m}$

Sol.

$$M = \frac{3}{4} V_B d_L$$

$$V_B d_B = \frac{75}{100} V_B \times 1000$$

$$d_B = 750$$

$$M + m = V_B d_L$$

$$(~~M+m~~) \quad V_B d_B = V_B \times 1000$$

$$\Rightarrow \quad \frac{M+m}{M} = \frac{4}{3}$$

$$\Rightarrow \quad 3M + 3m = 4M$$

$$\Rightarrow \quad 3m = M$$

$$\Rightarrow \quad \boxed{\frac{M}{m} = \frac{3}{1}}$$

Que1. Density of sea water & ice berg are 1100 Kg/m³ & 920 Kg/m³ respt. find % volume of ice berg above sea level.

Sol.

~~$V_i d_L g$~~

$$V_{in} d_L = V_B d_B$$

$$\frac{V_{in}}{V_B} = \frac{d_B}{d_L} = \frac{920}{1100}$$

$$\% \text{ inside} = \frac{920}{110} \times 100$$

$$= 83.6$$

$$\% \text{ outside} = 16.3\%$$

$$V_i d_L g = V_s d_s g$$

$$\frac{V_i}{V_{sh}} = \frac{100}{92} - 1$$

$$V_i - \frac{100-92}{92} \times 100$$

Que. A boat of 600kg is floating in lake with 7cm submerged Height when a man enters in it, submerged Height becomes 8.4 cm Mass of Man =?

$V_{in} = \frac{7}{L} V_B$

sol.

$$600 = (A \times h_1)\, d_L$$

$$600+m = A h_2\, d_L$$

$$\frac{600+m}{600} = \frac{h_2}{h_1}$$

$$\frac{600+m}{600} = \frac{8.4}{7}$$

$$\boxed{m = 120\ Kg}$$

$M_B = V_B d_B = V_{in} d_L$

$600 = \frac{7}{L} V_B d_L$

$600+m = \frac{8.4\, V_B}{L}$

$720 = 600+m$

Que. A Man enters in a boat of Area 3m x 2m, then submerged Height ↑es by 1cm. Mass of man =?

sol.

$$M = A h_1 d_L$$

$$M+m = A h_2 d_L.$$

$$m = A(h_2 - h_1)\, d_L.$$

$$m = 6(1 \times 10^{-2})(10^3)$$

$$\boxed{m = 60\ Kg}$$

$(M+m) = 3 \times 2 \cdot 01$

$(M+m) = 6.01$

$\frac{M}{M+m} = \frac{6}{6.01}$

Que. Weight of crown in air = 210gm & in water = 198g It is made by copper + Gold, then find mass of Gold in it

$\rho_{Au} = 19.3\ g/cm^3$ $\qquad \rho_{cu} = 8.5\ g/cm^3$

sol.

$w_u + w_c = 210.$

$$\text{density of crown} = \frac{w_a}{w_a - w_w}\rho_w.$$

$$= \frac{210}{210-198} \times 1\ g/cm^3$$

$$= \frac{210}{12}\ g/cm^3$$

$B = 12$

$w_u + w_g = 192.$

$$\Rightarrow \quad \rho_{mix} = \frac{M_1 + M_2}{\left[\frac{M_1}{\rho_1} + \frac{M_2}{\rho_2}\right]} = \frac{m + (210 - m)}{\frac{m}{19.3} + \frac{210-m}{8.5}} = \frac{210}{12}.$$

$\cancel{12m + 12(210)}$

$$\Rightarrow \quad 12 = \frac{m}{19.3} + \frac{210-m}{8.5}$$

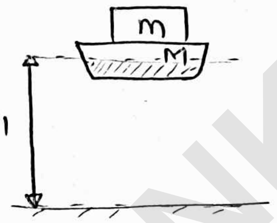
$$\Rightarrow \quad 12 \times 19.3 \times 8.5 = 8.5m + 19.3(210 - m).$$

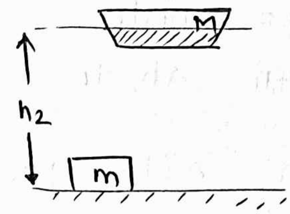
$$\boxed{m = 193\,g}$$

\# Question Based on water level.

Ques. A Heavy box is kept on a boat which floats on a lake. It falls down in the lake then discuss the water level on that place.

Sol.





Initially $Ah_1 = V_w + \left(\frac{M+m}{d_L}\right)$ → vol. of water displaced.

finally ⇒ $Ah_2 = V_w + \frac{M}{d_L} + \frac{m}{d_B}$

$$Ah_1 - Ah_2 = \frac{m}{d_L} - \frac{m}{d_B}$$

$$\boxed{Ah_1 - Ah_2 = m\left[\frac{d_B - d_L}{d_B\, d_L}\right]}$$

if

1. $d_B > d_L$ so $h_1 > h_2$ level will fall
2. $d_B = d_L$ so $h_1 = h_2$ level unchanged
3. $d_B < d_L$ so $h_1 < h_2$ levels rise.

Ques. When a ice cube with Metal balls embedded in it melts then water level =?

Sol. Fall $A h_1 - A h_2 = M\left(\frac{d_B - d_L}{d_B d_L}\right)$

when metal balls sink after melting of ice.

$A h_1 - A h_2 = M \frac{d_B - d_L}{d_B \cdot d_L}$

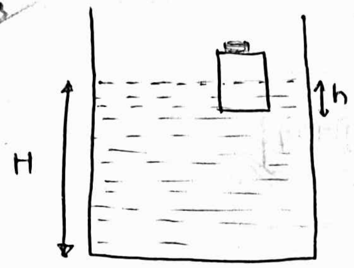
as $d_B > d_L$ here.

$h_1 > h_2$ → water level fall.

Ques. when a ice cube melts in glass of water then?

Sol. unchanged.

Ques.
Jee Mains



Metal coin is Kept on a wooden cube. when coil falls in container then.

① H & h ↑es.

② H & h ↓es

③ H ↑ but h ↓es

④ H ↓ but h ↑es.

FLUID DYNAMICS

* Study of Fluid in Motion.

* Ideal fluid
  1. Incompressible
  2. Non viscous
  3. Stream line flow / laminar flow.
  4. Irrotational flow.

CONTINUITY EQUATION :-

* Based on mass conservatⁿ
* Mass of fluid incoming = Mass of Fluid outgoing

* Mass conservatⁿ :→

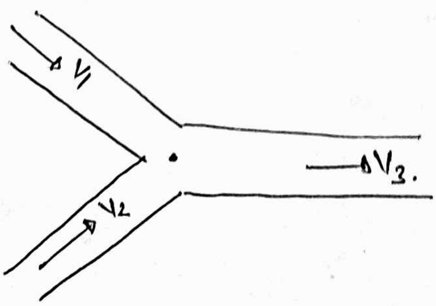
$M = \rho V \Rightarrow M = \rho (A l) \Rightarrow M = \rho A (V_0 t).$

$$\boxed{\frac{dM}{dt} = \rho A V_0}$$

1. $\frac{dM}{dt} = \text{const.}$
2. Quantity of flow $\boxed{Q = \frac{dV}{dt}}$ = rate of change of volume.

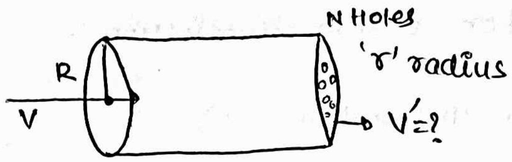
$A V_0 = \text{const.}$ → $V_0 \propto \frac{1}{A}$ → $\boxed{A_1 V_1 = A_2 V_2}$

Que.



$A_1 V_1 + A_2 V_2 = A_3 V_3.$

Ques.



$$(\pi R^2) V = N(\pi r^2) V'$$

$$\boxed{V' = \frac{V}{N}\left(\frac{R}{r}\right)^2}$$

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L-2 TL-77

* Energies of a Flowing liquid :-

* Kinetic Energy :- Due to the motion of liquid particles

$$\boxed{K.E = \frac{1}{2} M V_0^2}$$

$$\boxed{K.E = \frac{1}{2}(\rho V) V_0^2}$$

- K.E / unit vol. $\boxed{\frac{K.E}{V} = \frac{1}{2}\rho V_0^2}$

- K.E / unit weight $= \frac{V^2}{2g}$ = Kinetic Head.

* Potential Energy :- Due to position w.r.t reference.

- $\boxed{U = mgh = \rho V g h.}$

- P.E / unit volume $\boxed{\frac{U}{V} = \rho g h}$

- P.E / unit weight = h = Potential Head or Gravitational Head.

* Pressure Energy :- Due to pressure

- $\boxed{P.E = PV}$

- PE / unit vol. $\boxed{\frac{P.E}{V} = P}$

- PE / unit weight $= \frac{P}{\rho g}$ = Pressure Head.

BERNOUILLIS EQUATION :- ⇒ Based on Energy conservation.

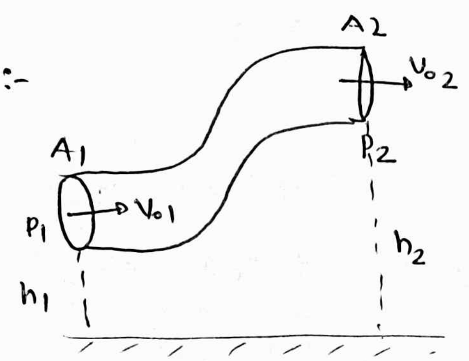
Acc. to it sum of K.E, P.E & ~~P~~ P.E per unit volume for an ideal fluid is constant.

* $$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{const}} \Rightarrow \text{per unit vol.}$$

$$\boxed{\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{const.}} \Rightarrow \text{per unit weight.}$$

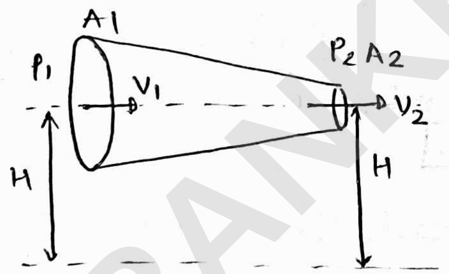
⇒ $$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2}$$

Example:-



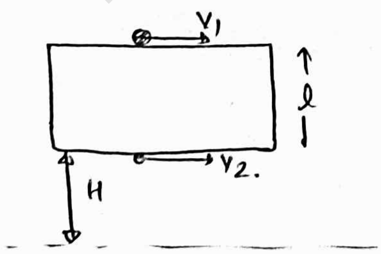
* $A_1V_1 = A_2V_2$
* $P_1 + \frac{1}{2}\rho V_{01}^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_{02}^2 + \rho g h_2$

Ex:-



* $A_1V_1 = A_2V_2$
* $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

Ex:-



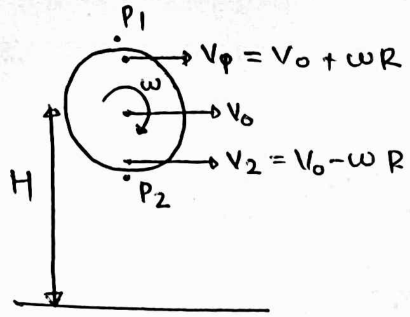
* $P_1 + \frac{1}{2}\rho V_1^2 + \rho g(H+l) = P_2 + \frac{1}{2}\rho V_2^2 + \rho g H$
* if $l \ll H$.

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$P + \frac{v^2}{2g} = C - h$ $\quad y = mx + c$

$P = C - h - \frac{v^2}{2g}$ $\quad A_1V_1 = A_2V_2$

Ques.



* $P_1 + \frac{1}{2}\rho(V_1^2) = P_2 + \frac{1}{2}\rho(V_2^2)$

* $P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2) = \frac{1}{2}\rho(4V_0\omega R)$

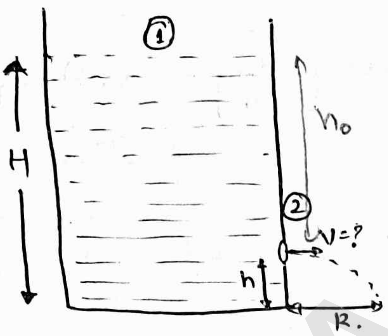
* $\boxed{P_2 - P_1 = 2\rho V_0 \omega R}$

eg:- Base ball
cricket volleyball
Football

MAGNUS EFFECT.

TORICELLI'S EQUATION :-

* spd. of efflux :-



$\Rightarrow P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2.$

$\Rightarrow \frac{1}{2}\rho V_1^2 + \rho g H = \frac{1}{2}\rho V_2^2 + \rho g h$

$$\boxed{V_2^2 = V_1^2 + 2g(H-h)}$$

* Top layer at Rest $V_1 = 0$

$\cancel{V_2 = 2g\sqrt{h}}$

$$\boxed{V_2 = \sqrt{2g(H-h)}} \quad \boxed{V_2 = \sqrt{2gh_0}}$$

(*) if the hole is at depth 'h' from top surface.

$$V = \sqrt{2gh}.$$

* Range $\Rightarrow$ $R = ut$

$u = \sqrt{2gh}$

$t = \sqrt{\frac{2(H-h)}{g}}$

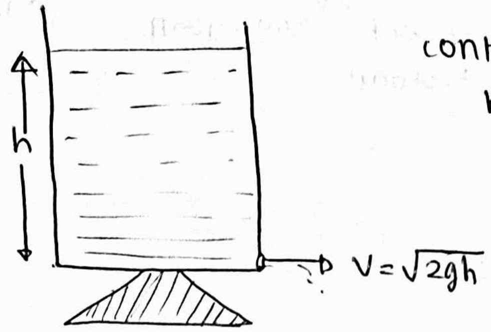
$$\boxed{R = \sqrt{4h(H-h)}}$$

* condition to get max Range.

* $\frac{dR}{dh} = 0$ $h = \frac{H}{2}$

* $\boxed{R_{max} = H}$

Ques.



container की Radius = R
hole की Radius = r

time to empty = ?

* $A_1V_1 = A_2V_2$

$\pi R^2 \left[-\frac{dh}{dt}\right] = (\pi r^2)\sqrt{2gh}$

$\Rightarrow \int_h^0 \frac{-R^2}{r^2\sqrt{2g}}\left[\frac{dh}{\sqrt{h}}\right] = \int_0^t dt$

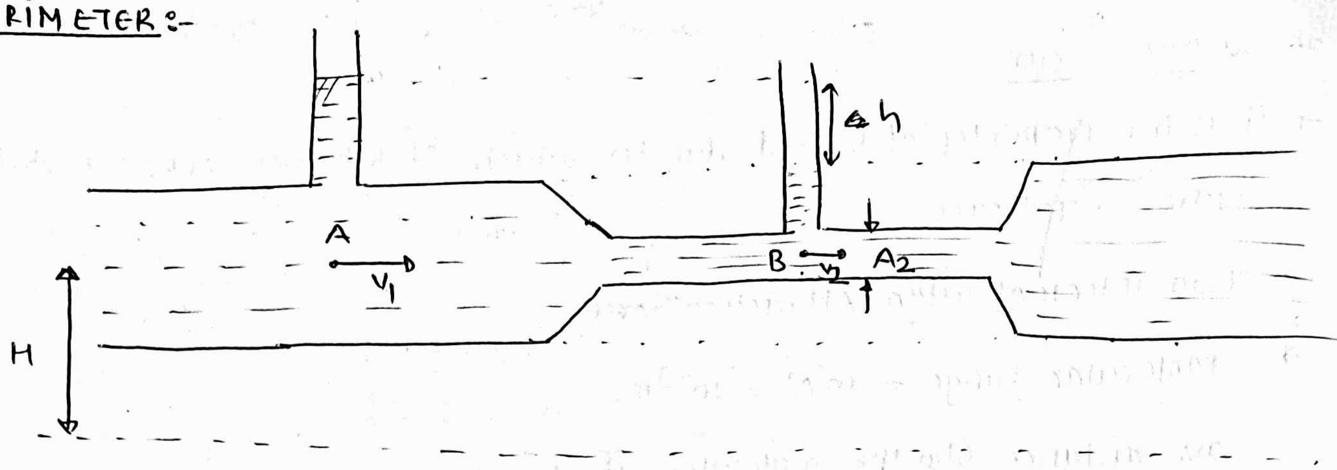
$\Rightarrow t = \frac{-R^2}{r^2\sqrt{2g}}\left(2\sqrt{h}\right)_h^0$

$\Rightarrow \boxed{t = \frac{2R^2\sqrt{h}}{r^2\sqrt{2g}}}$

$\Rightarrow \boxed{t = \frac{2A_1}{A_2\sqrt{2g}}(\sqrt{h})}$

$\Rightarrow \boxed{t = \frac{A_1}{A_2}\sqrt{\frac{2h}{g}}}$

VENTURIMETER :-



$$A_1V_1 = A_2V_2 = Q$$

$$P_A = P_0 + \rho g(h+x)$$

$$P_B = P_0 + \rho g x$$

* $P_A + \frac{1}{2}\rho V_1^2 = P_B + \frac{1}{2}\rho V_2^2$

* $P_A - P_B = \rho g h = \frac{1}{2}\rho(V_2^2 - V_1^2)$

$$\boxed{V_2^2 - V_1^2 = 2gh}$$

* $\boxed{V_1 = \frac{(\sqrt{2gh})\,A_2}{\sqrt{A_1^2 - A_2^2}}}$ $\boxed{V_2 = \frac{A_1\sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}}$ $\boxed{Q = \frac{A_1A_2\sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}}$

* Note:- $A_2 << A_1$

① $\boxed{V_1 = \frac{A_2}{A_1}\sqrt{2gh}}$

② $\boxed{V_2 = \sqrt{2gh}}$

③ $\boxed{Q = A_2\sqrt{2gh}}$

Surface Tension

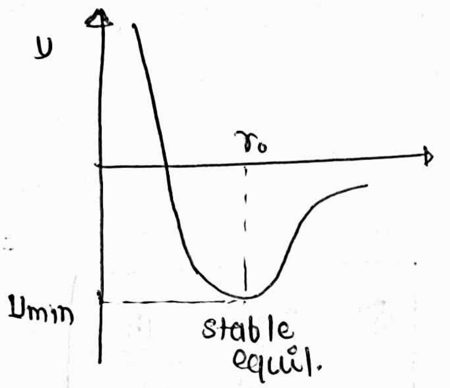
→ it is the property of liquid due to which it behaves like a ~~stret~~ stretched elastic membrane

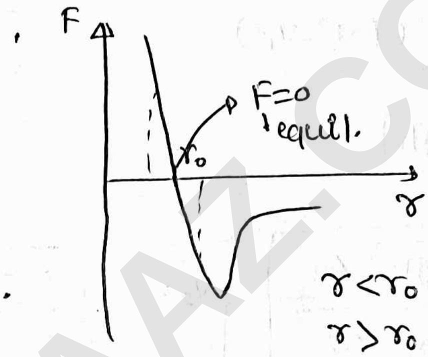
→ Two Intermolecular Attraction force.

* Molecular Range = 10 A° = 10^{-9}m.

av. distance b/w the molecules = if ↑es then intermolecular forces rapidly ↓es.

$$\boxed{U = \frac{A}{r^{12}} - \frac{B}{r^6}}$$





$r < r_0$ ⟹ IMF ⟹ Repulsive

$r > r_0$ ⟹ IMF ⟹ Attractive

① Adhesive forces

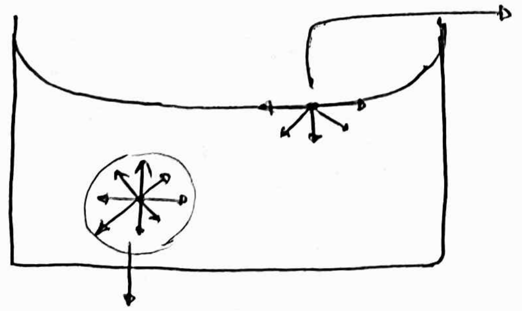
Two diff matter के Molecules के बीच का IM Attraction.

Ex. Liquid + wall of container.

② cohesive forces

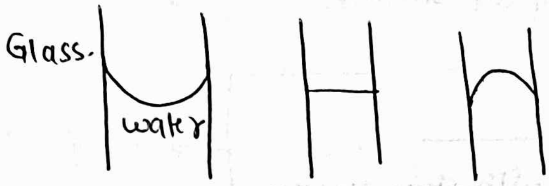
forces b/w molecules of same material.

*



sphere of Molecular activity. = radius of the order of Molecular range.

* surface has the property to contract itself
↓
to minimise P.E
↓
To gain stable config. Again.



Ex:-

① ~~Drop~~ ⇒ small drops are spherical in shape.

② in water Hair of brush spreads out but when it is taken out they stick to each other.

③ it is used to glue two surface with the help a liquid.

④ Hot soup, coffee, tea taste better as compared to cold ones.

⑤ Hot water mixed with detergent can clean dirty clothes easier as compared to ~~plane~~ plain water.

☆ Dependence of surface Tension

① $\boxed{\text{on temperature}}$ (T_0)

• By ↑sing the temp. of a liq., surface tension ↓es.

$$\boxed{T \propto \frac{1}{T_0}}$$

② $\boxed{\text{on impurities}}$

* Partially soluble impurities

→ S.T ↓es.

* completely soluble impurities

→ S.T ↑es

③ $\boxed{\text{contamination}}$

when on liq. surface dust particle or lubricating materials stays then it is cla "contamination of liquid" which ↓es the surface Tension.

surface Tension

* S.T in terms of force

→ Force per unit length of ~~an~~ ~~imagi~~ acting on an imaginary line at right Angles to it. & in plane of liquid surface.

$$\boxed{T = \frac{F_{net}}{\text{Total length in contact with liq surface}}}$$

Ex.

length = $2l$

force due to S.T $\boxed{F = T(2l)}$

Ex. Ring (circular frame of wire)

$L = 2(2\pi r)$

$F = T(4\pi r)$

Ex. Disc

$L = 2\pi r$

$F = T(2\pi r)$

Ex.

$L = 2\pi(r_1 + r_2)$

$F = T[2\pi(r_1 + r_2)]$

Ex. square plate

$L = 4a$

$\boxed{F = T(4a)}$

SURFACE ENERGY

* Additional energy due to the position and configuration of molecules of liquid surface.

$$\boxed{S.E = T \times A}$$

$S.E = T \times \text{Area} \Rightarrow$ in contact with Air.

* surface Tension = $T = \dfrac{dU}{dA} = \dfrac{\Delta U}{\Delta A} = \dfrac{W}{\Delta A}$

i.e $\underbrace{\text{W.D to expand}}$ surface by unity.

$\Downarrow$

change in surface energy $= U_f - U_i$

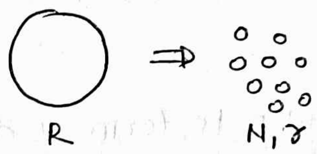
*

for $Q \to *$ $\Delta Q = 0$

surface Area ↑es.
↓
surface energy ↑es ($W = +ve$)
↓
Internal energy ↓es
↓
Temp. ↓es.

पानी in cup
↓ To be done cold
↓
Plate में Pour.

*



$$\boxed{R^3 = N r^3}$$

Ex:- liquid surface.

① surface = 1

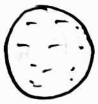
(2) Area = A
- → reactangle – lb
- → circle – πr^2
- → square – a^2

Ex. liquid film (soap film)

(1) surface = 2

(2) Area
- → $2(lb)$
- → $2(\pi r^2)$
- → $2(a^2)$

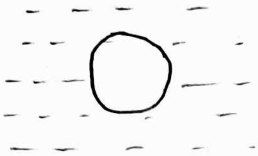
Ex. Liquid drop in air.



(1) surface = 1

(2) Area = $4\pi r^2$

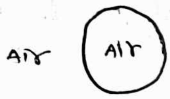
Ex. Air bubble in liquid.



(1) surface = 1

(2) Area = $4\pi r^2$

Ex. soap-bubble in Air



(1) surface = 2

(2) $A = 2(4\pi R^2)$

Ex. W.D to form a drop of 'R' S.T = T

$$W = T(4\pi R^2)$$

Ex. W.D to form a drop of r_2 from r_1

OR

To expand from r_1 to r_2

$$W = T(4\pi r_2^2 - 4\pi r_1^2)$$

Ques. if W is the work done to form a bubble of volume 'V'. find work done in forming the bubble of vol. '2V' ~~from the~~ from the same soap solu^n?

Sol

$T = \frac{W}{A}$

$W = T\,2(4\pi R^2)$

$$V = \frac{4}{3}\pi R^3$$

$$2V = \frac{4}{3}\pi R_1^3$$

$$\therefore \quad R_1^3 = 2R^3$$

$$\boxed{R_1 = 2^{1/3} R}$$

$$W = T(4\pi R^2) \times 2$$

$$W' = T(4\pi R_1^2) \times 2.$$

$$\frac{W'}{W} = \left(\frac{R_1}{R_2}\right)^2.$$

$$\boxed{W' = 2^{2/3} W}$$

Ques. A water drop of radius = 1mm is divided in 10^6 small droplets $T_w = 72$ dyne/cm the find the energy spent in the process.

sol.

$$R^3 = N r^3$$

$$R^3 = 10^6 r^3$$

$$\boxed{R = 10^2 r}$$

$$E_i = T(4\pi R^2) = T4\pi R^2$$

$$E_f = N[T(4\pi r^2)] = 10^6\left(4\pi T \frac{R^2}{10^4}\right) = 100\,4\pi R^2 T$$

$$E_f - E_i = 100\,4\pi R^2 T - 1\,4\pi R^2 T$$

$$= 99\,4\pi R^2 T$$

$\frac{4}{3}\pi R_1^3 = N$

$10^{-9} = 10^6 R^3$

$R^3 = 10^{-3}$

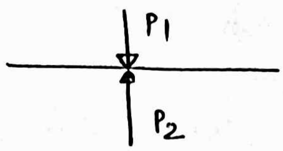
$S.E = 72$

72×10^{-3}

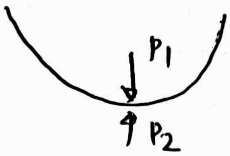
$72 \times 10^{-3} \times 2 \times 4\pi \times 10^{-6}$

$72 \times 10^{-3} \times 2 \times 4 \times \pi \times 10^{-5}$

* Excess pressure inside a curved surface :-



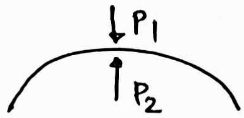
$\Rightarrow P_1 > P_2$



P_{excess} = Net pressure on the surface

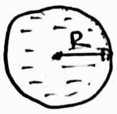
$= P_1 - P_2$

$\Rightarrow P_2 > P_1$



$P_{excess} = P_2 - P_1$

Ex. In a Drop in air

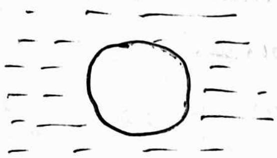


$P_{excess} = P_{in} - P_{out} = \frac{2T}{R}$

$\boxed{P_{in} = P_{out} + \frac{2T}{R}}$

$P_{out} = P_0$

Ex. In Air Bubble in liquid



$P_{excess} = P_{out} + \frac{2T}{R} = P_{in}$

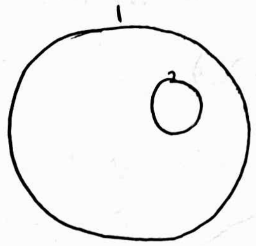
Ex. soap bubble in air

Two surfaces

$P_{in} - P_{out} = \frac{4T}{R}$

$\boxed{P_{in} = P_{out} + \frac{4T}{R}}$

* Double Air Bubble



$r_1 > r_2$

$\Rightarrow \quad P_{in_2} - P_{out_2} = \frac{4T}{r_2}$

$\Rightarrow \quad P_{in_1} - P_{out_1} = \frac{4T}{r_1}$

$\Rightarrow \quad \boxed{P_{in_1} = P_0 + \frac{4T}{r_1}}$

$\Rightarrow \quad P_{in_2} = P_{out_2} + \frac{4T}{r_2}$

$\Rightarrow \quad P_{in_2} = P_{in_1} + \frac{4T}{r_2}$

$\Rightarrow \quad \boxed{P_{in_2} = P_0 + \frac{4T}{r_1} + \frac{4T}{r_2}}$

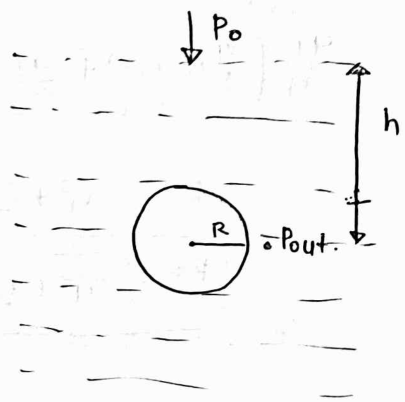
* Air Bubble At some depth from the surface :-

$P_{in} - P_{out} = P_{excess}$

$P_{in} = P_{out} + P_{excess}$

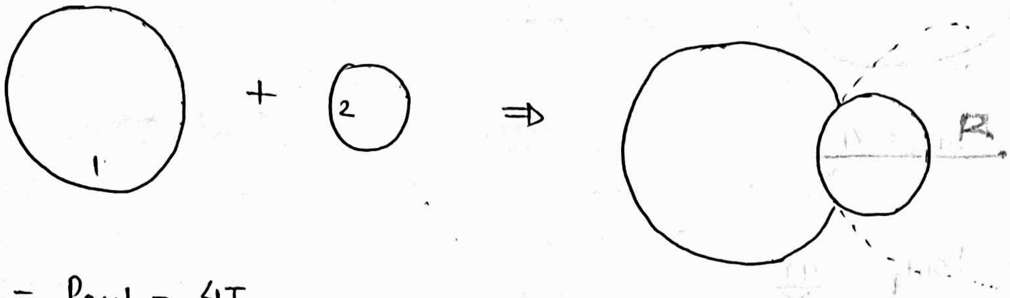
$P_{out} = P_0 + \rho g h$

$\boxed{P_{in} = P_0 + \rho g h + \frac{2T}{R}}$



Ques. Two soap bubbles of radii r_1 + r_2 come in contact in air. find the radius of curvature at point of contact? (R) $(r_1 > r_2)$

Sol.



$$P_{in_1} - P_{out} = \frac{4T}{r_1}$$

$$P_{in_2} - P_{out} = \frac{4T}{r_2}$$

$$\underbrace{P_{in_2} - P_{in_1}}_{\Downarrow} = 4T\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$\frac{4T}{R} = 4T\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$\boxed{R = \frac{r_1 r_2}{r_1 - r_2}}$$

Ques. Two soap ~~so~~ bubble of radii r_1 + r_2 combine to form a bigger bubble Isothermally. Radius of resultant bubble = ?

Sol.

$$P_1V_1 + P_2V_2 = PV$$

$$\frac{4T}{r_1}\left(\frac{4}{3}\pi r_1^3\right) + \frac{4T}{r_2}\left[\frac{4}{3}\pi r_2^3\right] = \frac{4T}{R}\left[\frac{4}{3}\pi R^3\right]$$

$$R^2 = r_1^2 + r_2^2$$

$$\boxed{R = \sqrt{r_1^2 + r_2^2}}$$

r_1 r_2

$P_{in_1} - P_{out} = \frac{4T}{r_1}$

$P_{in_2} - P_{out} = \frac{4T}{r_2}$

$P_3 - P_{out} = \frac{4T}{r_3}$

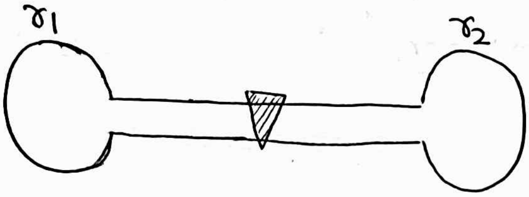
$P_{in_2} - P_{in_1} = 4T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$\frac{4T}{r_1 + r_2} = 4T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$\frac{1}{r_1 + r_2} = \frac{(r_2 - r_1)}{r_1 r_2}$

$\frac{r_1 r_2}{r_1 - r_2} = r_3$

Ques.



$r_1 > r_2$

what will be the effect on size of the bubble when stop cork is removed?

Sol.

as $r_1 > r_2$

$(P_{in_1}) < (P_{in_2}) \Rightarrow$ Air will flow from Higher pressure to lower pressure

$P_{in_1} - P_{out} = \frac{4T}{r_1}$

$P_{in_2} - P_{out} = \frac{4T}{r_2}$

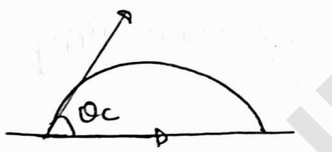
$P_{in_2} - P_{in_1} = 4T\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

Hence, Bigger bubble will get more bigger.

ANGLE OF CONTACT

* It is the Angle b/w planes. (Not lines)
* Angle b/w tangential plane at the liquid surface & the tangential plane at solid surface inside the liquid at point of contact.

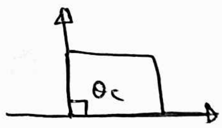
① Shape of drop

* 

$\theta_c < 90$

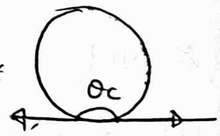
* 

$\theta_c > 90$

* 

$\theta_c = 90$

$\underbrace{\theta_c > 90 \quad \theta_c = 90}$ do not wet the surface

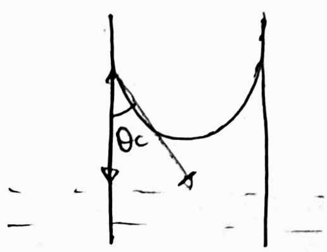
* 

$\theta_c = 180^\circ$ (lotus effect)

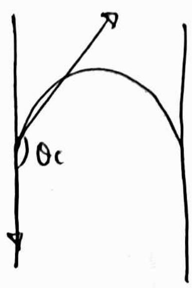
* 

$\theta = 0^\circ$

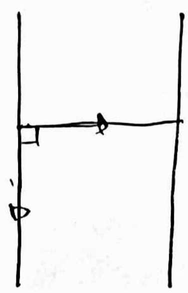
② Shape of Meniscus :- Meniscus $\Rightarrow$ shape of surface of liquid inside a capillary



$\theta_c < 90^\circ$



$\theta_c > 90^\circ$



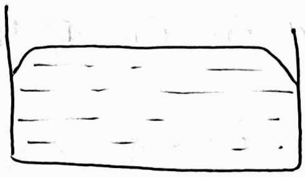
$\theta_c = 90^\circ$

③ Shape of liquid surface in a container:-

$\cancel{\theta_c >}$

$\theta_c < 90°$

(concave)



$\theta_c > 90°$

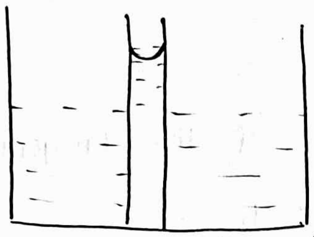
(convex)

$\theta_c = 90°$

(Plane)

④ Rise / fall of the liquid inside a capillary:-

$F_A > \frac{F_C}{\sqrt{2}}$



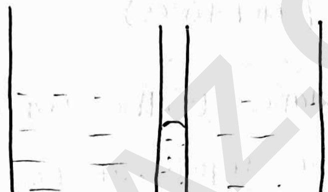
$\theta_c < 90°$

~~ex~~.

Rise

wets the surface

$F_A < F_C/\sqrt{2}$

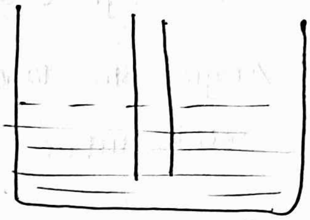


$\theta_c > 90°$

Fall

Does'nt wet the surface

$F_A = F_C/\sqrt{2}$



$\theta_c = 90°$

No change

No wetting

★ **Note :-**

① Angle of contact depends on liquid-solid pair.

② It does not depends on inclination of capillary in the liquid.

③ Angle of contact depends on surface Tension.

⇒ S.T of liquid ↑es ⇒ Angle of contact ↑es.

* All the factors Abbecting S.T can effect angle of contact similarly.

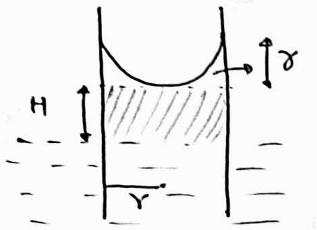
CAPAÎLLARITY :-

* <u>capâllary</u> :- cyUinder cal ~~Action~~ glass tube of small radius.

* <u>capillarity</u> :- Property of liq. due to which it rises or fall in capillary tube.

* <u>capillary Action</u> :- Phenomenon of rising or depressing the level of liq. inside capillary.

*



radius of capillary $= r$

volume $V = \pi r^2 h + \left(\pi r^2 . r - \frac{\pi r^3}{3} \right)$

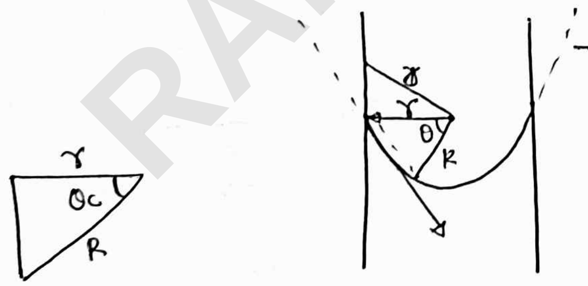
$$\boxed{V = \pi r^2 \left(h + \frac{r}{3} \right)}$$

<u>correction</u> :-

$$v' = \frac{\pi r^3}{3}$$

Mass rises $= \rho(\pi r^2 h)$

*



→ Radius of curvature of the meniscus.

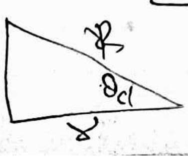
$$\boxed{\cos\theta_c = \frac{r}{R}}$$

$$\frac{2T}{R} = \rho h g$$

$$\boxed{h = \frac{2T}{R\rho g}}$$ (in terms of R)

$$\boxed{\theta \propto \frac{1}{\rho}}$$

$$\boxed{h = \frac{2T\cos\theta_c}{r\rho g}}$$ (in terms of r)



* For a particular solid- liq. pair

$T, \theta_c, \rho =$ fixed

$\boxed{h \propto \frac{1}{r}}$ (zurin's law)

* $\cos \theta_c$ $\quad$ 0° to $90^\circ \Rightarrow$ ↑es $\quad$ $\cos\theta_c =$ ↓es $= +ve$

90° to $180^\circ \Rightarrow \theta_c =$ ↑es $\quad$ $\cos\theta_c =$ ↓es $= -ve$

h rise $\quad$ when $\theta_c \rightarrow$ $\quad 0^\circ \leq \theta_c < 90^\circ$

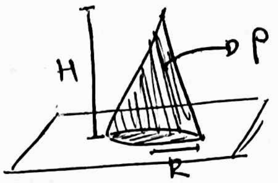
'h' falls $\quad$ when $\theta_c \rightarrow$ $\quad 90^\circ < \theta_c < 180^\circ$

* $h \propto \frac{1}{R}$ $\quad$ capillary of insufficient length is used

liq. will not come out as a ~~function~~ fountain but it will start to ↑es it radius of curvature to maintain

$\boxed{h_1 R_1 = h_2 R_2}$

Ques.



Find Total ~~Pressure~~ force exerted by water on cone?

Sol.

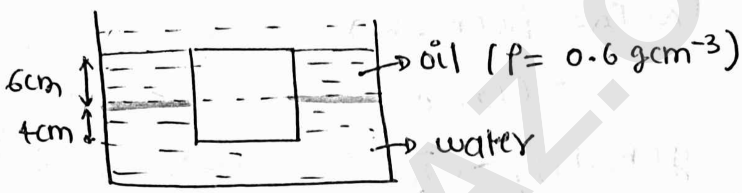
$$\boxed{F = PA}$$

$$F + Mg = \rho g H \pi R^2$$

$$F + \rho\left(\frac{1}{3}\pi R^2 H\right)g = \rho g H \pi R^2$$

$$\boxed{F = \frac{2}{3}\rho g H \pi R^2}$$

Ques.



edge length of block = 10cm.

find Mass of block

Sol.

$$Mg = \rho V g$$

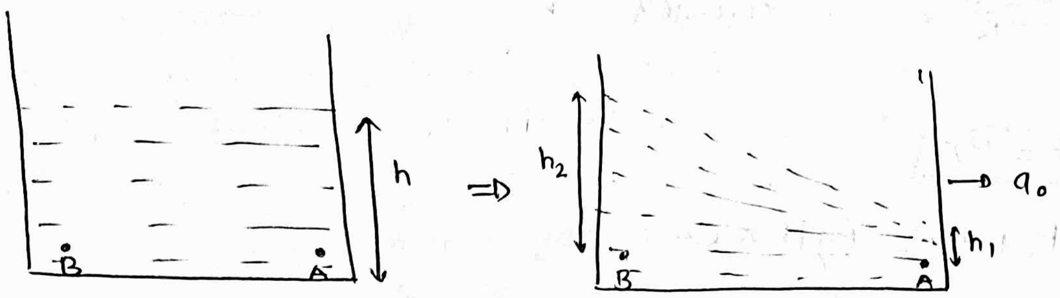
$$M = \rho_{oil}\, h_1 \times A_1 + \rho_w\, h_2 A_2$$

$$= 0.6 \times 6 \times 10^2 + 1 \times 4 \times 10^2$$

$$\Rightarrow [3.6 + 4]\, 10^2$$

$$\boxed{M = 760\ gm}$$

FOR ACC. FLUIDS



$P_A = P_B = P_0 + \rho g h$

$P_A = P_0 + \rho g h_1$

$P_B = P_0 + \rho g h_2$

$P_B = P_A + \rho a_0 l \quad \left(ma = \frac{(\rho A l) a_0}{A} = \rho a_0 l \right.$

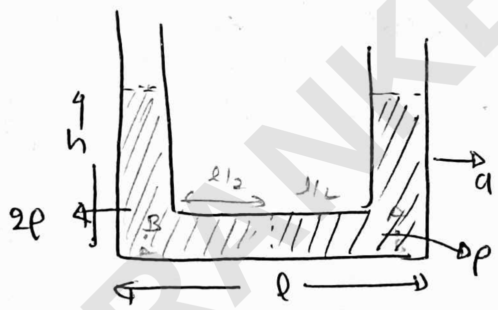
$P_B - P_A = \rho g (h_2 - h_1)$

$= \rho l a_0$

$g(h_2 - h_1) = l a_0$

$$\boxed{\frac{h_2 - h_1}{l} = \frac{a_0}{g} = \tan\theta}$$

Ques. PYQ



In a open yoututbe equal vol. of 2 liquids are kept
what must be $\underline{a}$ so that ht. diff b/w liquid surface become zero

Ans.

$P_A = P_0 + \rho g h$

$P_B = P_0 + 2\rho g h$

$P_B - P_A = \rho g h$

$P_B = P_A + \rho a_0 \frac{l}{2}$

$P_B = P_A + (\rho_1 + \rho_2) a_0 \frac{l}{2}$

$\rho g h = 3\rho a_0 \frac{l}{2}$

$$\boxed{a_0 = \frac{2gh}{3l}}$$

$F = \rho n g$

$\Rightarrow \cancel{(\rho_1 + \rho_2) g \frac{l}{2} A}$

$P A$

$\Rightarrow (\rho_1 + \rho_2) g \frac{l}{2}$

$(2\rho - \rho)$

$F =$

$\frac{3g}{2}$

for $P_B = P_A + (\rho_1 + \rho_2) a_0 \frac{l}{2}$

$\Rightarrow$

$P_B - P_A = \rho g h$

$\rho g h = 3\rho a_0 \frac{l}{2}$

$a_0 = 2$

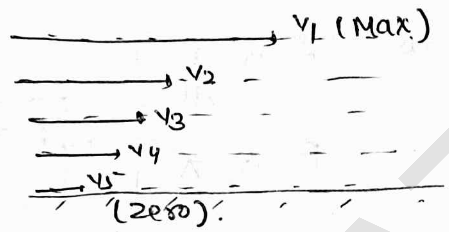
VISCOCITY

* It is ~~a~~ property of a fluid by which it can resist ~~their~~ its flow. that means property of a fluid by virtue of ~~its~~ which it opposes the motion (relative) b/w the adjacent layers inside the fluid.

* from observations we found more viscous liquid follow the steady or stream line or laminar flow.

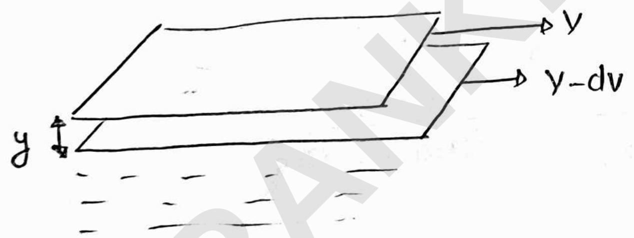
  less viscous liquids have the tendency to follow turbulent flow.

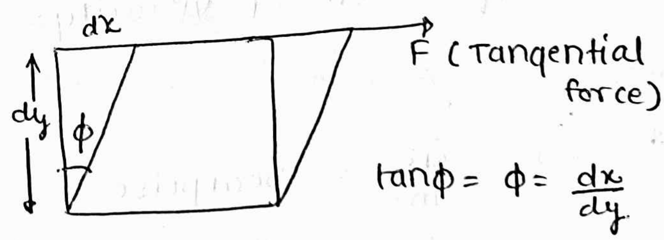
* from observations we found velocity of layers ↓es as depth from upper surface ~~↓e~~ ↑es.



$v_1 > v_2 > v_3 > v_4 > v_5$

* velocity Gradient :- $\frac{dv_x}{dy} = \frac{dv}{dh}$ = variation in speed of layers w.r.t depth / ↑ height.





$\tan\phi = \phi = \frac{dx}{dy}$

dx = additional dist. covered by upper layer b'coz of 'dv'

$$\boxed{dx = (dv)(dt)}$$

$x = vt$

* <u>velocity gradient = strain rate</u>

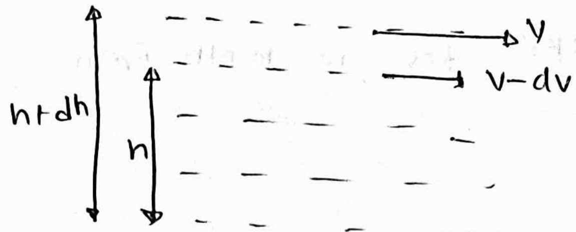
* shear stress = $\frac{F_{tan}}{A} = \eta$ (shear strain)

$\Rightarrow \phi =$ shear strain

$$= \frac{(dV_x)(dt)}{dy}$$

$$\boxed{\frac{\phi}{dt} = \text{strain-rate} = \frac{dV_x}{dy}}$$

* Newton's law of viscocity :-



$\Rightarrow$ tangential force is acting which opposses ther relative motion.

$\Rightarrow$ $F \propto \frac{dv}{dh}$, $F \propto A$ $\quad \boxed{F \propto A\left(\frac{dv}{dh}\right)}$

Tangential $\Rightarrow$ $\boxed{F = -\eta A \left(\frac{dv}{dh}\right)}$ $\quad$ −ve = spd./ Motion की dirⁿ के opp.

viscous force

$\eta =$ coeff. of viscocity $= \frac{F/A}{(dv/dh)} = \frac{\text{stress}}{\text{strain-rate}}$

* S.I = $\frac{Ns}{m^2}$ = Decapoise

$\boxed{1 \text{ Decapoise} = 10 \text{ Poise}}$

* C.G.S = $\frac{\text{dyne s}}{cm^2}$ = Poise

$\boxed{1 \text{ Poise} = 0.1 \text{ Decapoise}}$

* Dependency of viscocity

1. on temperature
   - for liquids $\Rightarrow$ $\eta \propto \frac{1}{\sqrt{T}}$ (cohesive forces ↓es)
   - for gases $\Rightarrow$ $\eta \propto \sqrt{T}$ (Diffusion ↑es)

2. on Pressure
   - for liquids $\Rightarrow$ As pressure ↑es, viscocity ↑es
   - for gases $\Rightarrow$ Does not depend

3. on Nature of fluid.

* Reynold's Number :-

→ This defines liquidity of fluid.

→ $\boxed{\text{liquidity} \propto \frac{1}{\text{viscocity}}}$

→ $\boxed{R = \frac{\rho v d}{\eta}}$

ρ = dencity
V = Avg. speed of molecules
d = diameter of the tube
η = vis cocity.

* when

$R < 1000$ $\Rightarrow$ laminar flow

$R > 2000$ $\Rightarrow$ Turbulent flow

$1000 < R < 2000$ $\Rightarrow$ can't say, ~~any thing~~ depend on other factors

* when fluid particles are in streamline motion, then ~~they~~ ↑es their spd. upto a certain value so that they can achieve turbulant flow. then this max spd. is c/a critical spd. for streamline flow

Que. In a river of depth 1 km, upper layer is moving with spd. 5m/sec find shear stress. $\eta_{water} = 0.01$ Poise.

Sol.

$h = 10^3 m.$

$v = 5$

$\eta = 10^{-3}$ D.P.

$$\text{sh. stress} = \eta\left(\frac{dv}{dh}\right) = 10^{-3} \times \frac{5}{10^3} = 5\times10^{-6}\ N/m^2.$$

$\frac{F}{A} = \eta$

$\eta = \frac{dv}{dh}$

Que. An oil layer has <u>thickness = 1mm</u> + it is kept b/w two Glass plates. (Area = 10cm²)
if diff in spds. = 4m/s. b/w Glass plates. when a <u>$F_{tan} = 10N$</u> applied on upper plate
Find $\eta_{oil} = ?$

Sol.

$$F = \eta A\left(\frac{dv}{dh}\right)$$

$$10 = \eta\,(10\times10^{-4})\,\frac{(4)}{10^{-3}}$$

$$\boxed{\eta = 2.5 \text{ Decapoise}}$$

$10 = x\,(\cancel{10}\times10^{-4})\times\frac{4}{10^{-3}}$

$x = \frac{10}{4} = 2.5.$

$\Rightarrow$ ~~2~~25 Poise

STOKES LAW

* when a spherical body is moving in Homogenous stationary viscous medium then A viscous force is applied on the body by the medium which is clq viscous force & it is given by

$$\boxed{F_v = 6\pi \underbrace{\eta}_{\text{viscosity}} \underbrace{r}_{\text{radius of spherical body}} \underbrace{V_T}_{\text{terminal velocity}}}$$

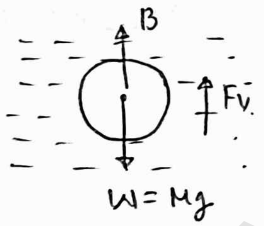
* Terminal velocity :- (V_T)

* it is the const. velocity Attained by a spherical body when falling through Any fluid

* const. velocity means → All the forces acting on body are balanced.

* $d_B > d_L \Rightarrow$ Motion $\Rightarrow$ downwards $= F_v = \uparrow$

  $d_L > d_B \Rightarrow$ Motion $\Rightarrow$ upwards $= F_v = \downarrow$



* when velocity const. = Terminal velocity

$\Rightarrow B + F_v = W$

$W = V_B d_B g = V d_B g$

$B = V_{in} d_L g = V d_L g$

~~$V d_L g + V d_B$~~

$\Rightarrow V d_L g + 6\pi\eta r V_T = V d_B g$

$\Rightarrow 6\pi\eta r V_T = V(d_B - d_L) g$

$$\Rightarrow \boxed{V_T = \frac{2}{9}\frac{r^2 g (d_B - d_L)}{\eta}}$$

* $d_B > d_L \Rightarrow V \downarrow$
* $d_B < d_L \Rightarrow V \uparrow$

Que. Neet 2016 when a spherical body is falling through a viscous medium, then Rate of Heat generatⁿ depends on radius of body as
(a) r^2 (b) r^3 (c) r^4 (d) r^5

Sol.

$$\text{Power} = F_v V_T$$

$$= 6\pi\eta r V_T \cdot V_T$$

$$= 6\pi\eta r V_T^2$$

$$\boxed{P \Rightarrow r^5}$$

$P = F \cdot V$

$\Rightarrow r V_T^2$

$\Rightarrow r^3 \times r^2 = r^5$

Que. A drop of water of radius = 0.015 mm is falling through air $\eta_{air} = 2\times10^{-5}$ $V_T = ?$

Sol.

$$r = 15\times10^{-6}$$

$$V_T = \frac{2r^2(d_B - d_L)g}{9\eta_{air}}$$

$$= \frac{\cancel{2}}{9}\times\frac{225\times10^{-12}\times10^{3}\times10}{\cancel{2}\times10^{-5}}$$

$$\Rightarrow 25\times10^{-2}$$

$\Rightarrow$

9) 225 (25
18
4

Que. 64 droplets of water are falling through air with spd. 1cm/s if they combine to form what is the V_T

Sol.

$$R^3 = N r^3$$

$$\boxed{R = 4r}$$

$$\boxed{V_T \propto r^2}$$

$$\frac{1}{x} = \frac{r^2}{(4r)^2}$$

$$\boxed{x = 16\,cm/s}$$

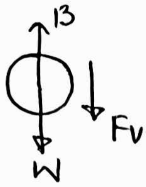
$R^3 = N r^3$

$R = 4r$

$\frac{1}{x} = \frac{r^2}{(4r)^2}$

Que. A Ball rises with const. velocity in a liquid ($d_L = 4d_B$) find the ratio of viscous force & weight.

Sol.



$$B = W + F_V$$

$$\frac{B}{W} = 1 + \frac{F_V}{W}$$

$V_T =$

$\frac{F}{W} = 6\pi\eta$

$$\Rightarrow \frac{F_V}{W} = 1 - \frac{B}{W}$$

$$\Rightarrow \frac{F_V}{W} = \frac{B}{W} - 1$$

$$= \frac{d_L}{d_B} - 1 = \frac{4d_B}{d_B} - 1 = 3$$