Date: 6 9 18

1. Motion In two Dimension

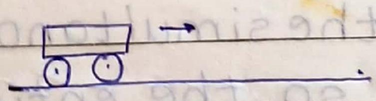
→ When a body moves such that any of its two co-ordinate changes with time hense such a motion is known as 2D motion.

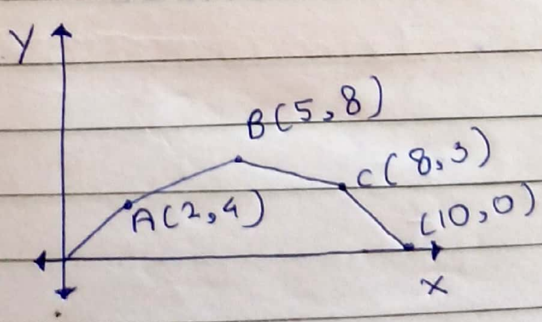
→ For ex: ➡ If a body is thrown at an angle θ with a horizontal then it will move in a parabolic path so such a motion is known as projectile motion.

→ In a projectile motion both of the co-ordinate (an (x, y)) changes with time so its an example of 2D motion.

→ To discuss this motion we will splited along the x-axis, along y-axis seperaltly

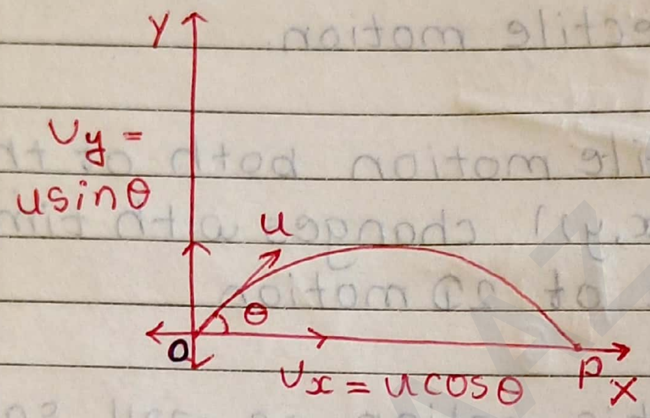
→ Example:

1D. 

2D. 

For the above discussion we can discussion projectile motion

* When a body is projected with an speed u at angle θ different result are as undee.



projectile motion 2D

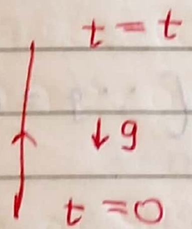
→ From the above Fig. it's cleae that path is a paeabola because body is moving undee the simultanous effect of $u\cos\theta$ & $u\sin\theta$ so the entiee motion canbe disscased as undee.

* projectile motion

X	Y
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_x = -g$
$v_x = u_x + a_x t \quad (\because a = 0)$	$v_y = v_x - gt$
$\therefore v_x = \underline{u_x = u\cos\theta}$.	$v_y = u\sin\theta - gt$

$$\therefore \underline{v_y = u\sin\theta - gt}.$$

* Time to reach x_{max}



$$t = \frac{u_y}{g} = \frac{u\sin\theta}{g}$$

* Total time of flight

$$t = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}.$$

* H max

$$v_y^2 = u_y^2 - 2gh$$

$$0 = u_y^2 - 2gh$$

$$h = \frac{u_y^2}{2g} = \frac{u^2 \sin^2\theta}{2g}$$

$$\therefore h_{max} = \frac{u^2 \sin^2\theta}{2g}$$

* Hz. Range.

$$R = u_x t + \frac{1}{2} g t^2 \quad (\because g = 0)$$

$$R = u_x t$$

$$= u_x \cdot \frac{2u\sin\theta}{g}$$

$$= \frac{u\cos\theta \cdot 2u\sin\theta}{g}$$

$$\therefore \text{Range} = \frac{u^2 \sin 2\theta}{g}$$

* Range is maximum at $\theta = 45°$

$$R_{max} = \frac{u^2}{g}$$

and $H_{max} = \cancel{t}\ \frac{R_{max}}{4}$

* Range is equal for θ & $(90-\theta)$

$$R_\theta = \frac{u^2 \sin 2\theta}{g} \quad — ①$$

$$R_{(90-\theta)} = \frac{u^2 \sin 2(90-\theta)}{g}$$

$= \frac{2u\sin\theta}{g}$ $= \frac{u^2\sin^2\theta}{g}$ $= \frac{u^2\sin^2\theta}{2g}$

$$= \frac{u^2 \sin(180-2\theta)}{g}$$

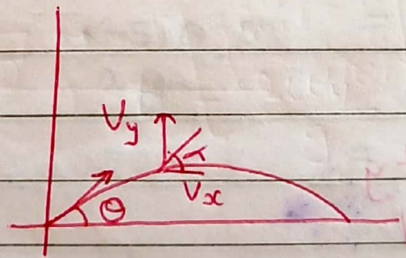
$\frac{u^2\sin^2\theta}{g}$

$$= \frac{u^2 \sin 2\theta}{g} \quad — ②$$

from ① & ②

$$R_\theta = R(90-\theta)$$

* Velocities at time T & direction of motion.



$$\vec{V} = V_x i + V_y j$$

$$\vec{V} = (u\cos\theta) i + (u\sin\theta - gt) j$$

* Speed

$$|V| = \sqrt{(u_x)^2 + (u_y - gt)^2}$$

* Direction

$$\tan\alpha = \frac{u_y}{V_x} = \frac{u_y - gt}{u_x}$$

$$\alpha = \tan^{-1}\left(\frac{u_y - gt}{V_x}\right)$$

If a body is projected at angle of 30° with verticle then find the following.

(i) Small t, T (Total time) $u = 100\,m/s$

(2) Hmax

(3) R

(4) Rmax

(5) Velocity

(6) Speed

(7) After 2sec.

(8) $m = 2kg$ find it K.E highest point

(1) $t = \dfrac{u\sin\theta}{g}$

$= \dfrac{u\sin 60^\circ}{10}$

$= \dfrac{\cancel{100}^{5}}{1\cancel{0}} \times \dfrac{\sqrt{3}}{\cancel{2}}$

$= 5\sqrt{3}$ sec.

(2) $T = 10\sqrt{3}$.

(3) $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{100 \times 100 \times 2 \times \sin 60^\circ \times \cos 60^\circ}{g}$

$= \dfrac{100 \times \cancel{100}^{5} \times \cancel{2} \times}{1\cancel{0} \times} \dfrac{\sqrt{3}}{\cancel{2}} \times \dfrac{1}{\cancel{2}}$

$= 100 \times 5 \times \sqrt{3}$

$= 500\sqrt{3}$

(4) $H_{max} = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{\cancel{100}^{25} \times \cancel{100}^{5} \times}{\cancel{2} \times 1\cancel{0}} \dfrac{3}{\cancel{4}}$

$= 25 \times 15$

$= 375$ m.

(5) Velocity afte 2sec =

$\vec{V} = u\cos\theta\, i + (u\sin\theta - gt) j$

$= \left(100 \times \dfrac{\cancel{\sqrt{3}}\,1}{2}\right) i + \left(100 \times \dfrac{\sqrt{3}}{2} - 20\right) j$

$$\vec{V} = 50i + 50\sqrt{3} - 20\ j$$

$$= \sqrt{(50)^2 + (50\sqrt{3}-20)^2}$$

$$= \sqrt{2500 + (50\sqrt{3})^2 - 2 \times 50\sqrt{3} \times 20 + 400}$$

$$= \sqrt{2500 + 2500 \times 3 + 400 - 40 \times 50\sqrt{3}}$$

$$= \sqrt{\cancel{5000}\ 2500 + 7500 + 400 - 40 \times 50\sqrt{3}}$$

$$= \sqrt{10000 + 400 - 2000\sqrt{3}}$$

$$= \sqrt{10400 - 2000\sqrt{3}}$$

$$= \sqrt{10400 - 2000 \times 1.73} = \sqrt{10400 - 3560}$$

(7) $K.E. = \frac{1}{2} \times (u\cos\theta)^2 \times m$

$$= \left(100 \times \frac{1}{2}\right)^2 \times 2 \times \frac{1}{2}$$

$$= \frac{100 \times \cancel{100}^{25}}{\cancel{4}} \times 2 \times \frac{1}{2}$$

$$= 2500 \times 2 \times \frac{1}{2}$$

$$= 5000\ \frac{1}{2}$$

$$= 2500$$

8. $R_{max} = \frac{u^2}{g} = \frac{100 \times 100}{10} = 1000 = 1 km$

Direction $= \frac{65}{50} = 1.3$

Q2 It a terririst fireing a bullet with a machine gun with welocity u then how much area will be in his range

$$\text{Area} = \pi R^2$$

$$= \pi \left(\frac{u^2}{g}\right)^2$$

$$= \frac{u^4}{g^2}\pi$$

3 On the planet of Jadu the velocity at time t is given by $v = ax\hat{i} + (b - ct)\hat{j}$ find the following.

(i) t
(ii) T
(iii) Hmax
(iv) R
(v) Rmax

$$v = ax\hat{i} + (b - ct)\hat{j}$$

$$0 = b - ct$$

① $\therefore t = \frac{b}{c}$

② $T = \frac{2b}{c}$

③ $u\sin\theta = b$

$accn. = c$

$\therefore h_{max} = \frac{b^2}{2c}$

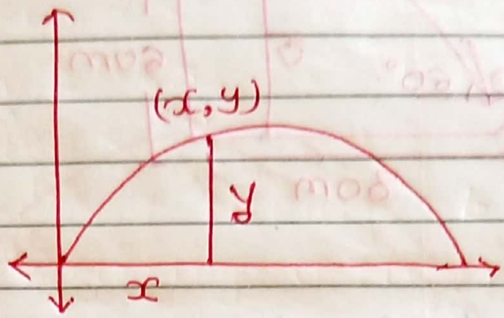
④. $Range = a_x \times t$

$= a_x \times \frac{2b}{c}$

$= \frac{2a_x b}{c} = \frac{2b^2}{c}$

$R = \frac{2u_x u_y}{g} = \frac{2ab}{c}$

* x and y co-ordinate & trajectory equation:



(1) $x = u_x t + \frac{1}{2} 0 t^2$

$x = u_x t$ — (1) $\qquad t = \frac{x}{u_x} = \frac{x}{u\cos\theta}$.

(2) $y = u_y t - \frac{1}{2} g t^2$

$y = (u\sin\theta)t - \frac{1}{2} g t^2$ — (2)

$= \cancel{u}\sin\theta \times \frac{x}{\cancel{u}\cos\theta} - \frac{1}{2} g \left(\frac{x}{u\cos\theta}\right)^2$

trajectory eq^n.

$$\boxed{y = x\tan\theta - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}}$$

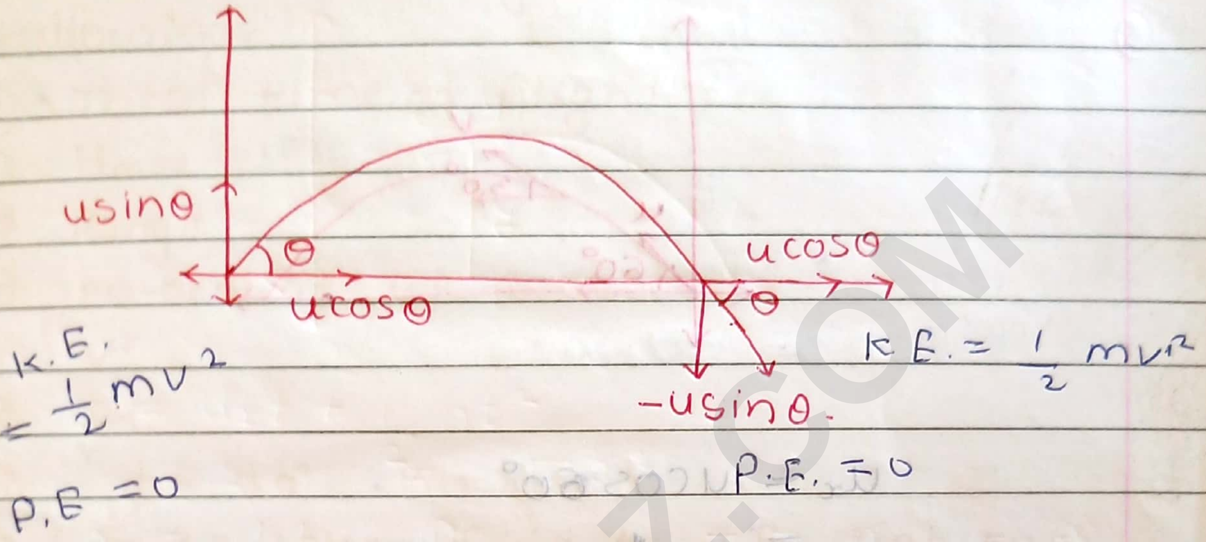
$y = x\tan\theta \left(1 - \frac{1}{2} \frac{g\,x}{u^2 \tan\theta \times \cos\theta}\right)$

$= x\tan\theta \left(1 - \frac{1}{2} \frac{g x}{u^2 \sin\theta \times \cos\theta}\right)$

$= x\tan\theta \left(1 - \frac{x}{\frac{u^2\, 2\sin\theta \times \cos\theta}{g}}\right)$

$$\boxed{y = x\tan\theta \left(1 - \frac{x}{R}\right)}$$

The velocity of hitting the surface.



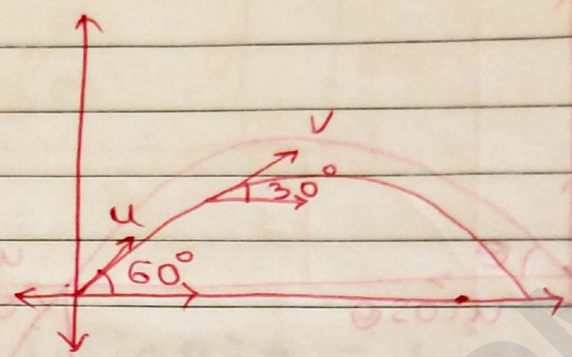
$$\frac{1}{\cancel{2}}\cancel{m}v^2 = \frac{1}{\cancel{2}}\cancel{m}v'^2$$

$$v^2 = v'^2$$

$$\boxed{v = v'}$$

When a body is projected with speed u. then it will strike the ground with the same speed. because the total energy emitts conserve.

Angle with horizontal when speed change:



$$u_x = u\cos 60^\circ$$
$$= u \times \frac{1}{2}$$

$$u_x = 50$$

$$v_x = v\cos 30^\circ$$

$$50 = v\frac{\sqrt{3}}{2}$$

$$v = \frac{100}{\sqrt{3}}$$

If a body is projected with intial speed $100\sqrt{2}$ m/s at an angle of 45°. then find the following.

1. Total time of flight
2. Hmax
3. Range.
4. co-ordinate at 2-sec.
5. v'
6. α.
7. $\phi = 30^\circ \quad v_1 = ?$
8. mass 2kg find its K.E = at top
9. velocity at ~~tee~~ 2sec. α
10. velocity at $\frac{1}{2}$ of maximum height.

③ $R = \frac{u^2}{g} = \frac{100 \times 10\cancel{0} \times 2}{1\cancel{0}} = \underline{\underline{2000m}}$

② $\frac{u^2}{4g} = \frac{2000}{4} = 5000m.$

① . $T = \frac{2u\sin\theta}{g} = \frac{2 \times 10\cancel{0} \times \sqrt{2}}{1\cancel{0} \times \sqrt{2}} = \frac{20}{\cancel{2}}$

$= 2\sqrt{2}$ sec

④ Co-ordinate at 2-sec.

$v = (u\cos\theta)\hat{i} + (u\sin\theta)^t \frac{1}{2}gt)$

$= \left(\frac{100\sqrt{2}}{\sqrt{2}}(2)\right)\hat{i} + \left(\frac{100}{\sqrt{2}}\right)^t - 10(2)\ \hat{j}$

$= (200\sqrt{2})\ \hat{i} + \left(\frac{100}{\sqrt{2}} - 20\right)\hat{j}$

(5)

(6) $\alpha = 45°$

(8) $K.E. = \frac{1}{2} m v^2$

$= \frac{1}{\cancel{2}} \times \cancel{2} \times (100\sqrt{2})^2$

$= 100 \times 100 \times 2$

$K.E. = 20000\ J$

(7) $\phi = 30°$ $\quad v_1 = ?$

$u_x = 100\sqrt{3} \cos 45°$

$= \frac{100\sqrt{3}}{\sqrt{2}}$

$= 50\sqrt{3} \times \sqrt{2}$

$\sqrt{2} \times 50\sqrt{3} = v \cos 30°$

$\frac{2 \times 50\cancel{\sqrt{3}} \times \sqrt{2}}{\cancel{\sqrt{3}}} = v \frac{\sqrt{3}}{2}$

$100\sqrt{2} = v$

(10) $v = 50\sqrt{2}$ & $-50\sqrt{2}$

$M.T.R. = \frac{u_y}{\sqrt{2}}$

$v = \sqrt{(100)^2 + \left(\frac{100}{\sqrt{2}}\right)^2}$

$= 100\left(\sqrt{1 + \frac{1}{2}}\right)$

$= 100\sqrt{\frac{3}{2}}$

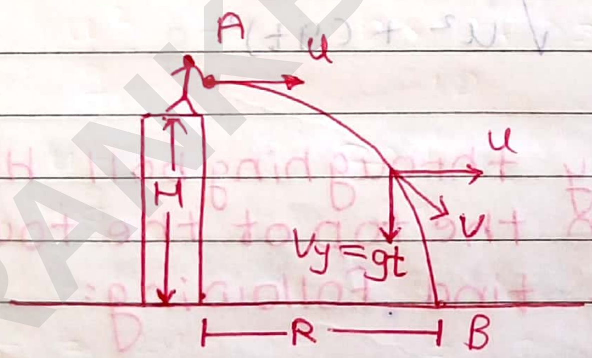
$$\text{No. of step} = \frac{2u^2h}{gb^2}$$

If we required velocity at certain height the x-compoenent remains constant but to calculate y-compoenent we will use the 3rd eqⁿ of motion. which will give us v_y thenwe we can calculate their speed by formula.

$$\sqrt{(V_x)^2 + (V_y)^2}$$

Projectile from height to ground:

When a body is projected from some height to the ground then these motion is known as projectile from height to ground.



$u_x = u$ —— ①

$u_y = 0$ —— ②

$a_x = 0$

$a_y = -g$

$$\text{Trajectory eq}^n\ y = \frac{1}{2} g \frac{x^2}{u^2}$$

$$H = \frac{1}{2} g t^2$$

$$t^2 = \frac{2H}{g}$$

$$\therefore t = \sqrt{\frac{2H}{g}}$$

$$R = u_x \times t$$

$$R = u \times \sqrt{\frac{2H}{g}}$$

Velocity after time =

$$\vec{V} = u\hat{i} + gt(-\hat{j}) = u\hat{i} + gt\,\hat{j}$$

$$|V| = \sqrt{u^2 + (gt)^2}$$

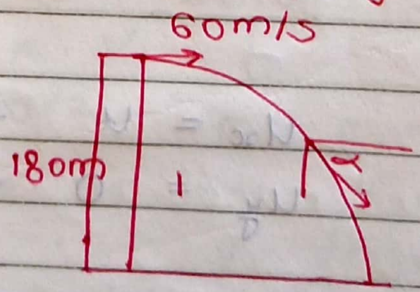
Q. A small boy throughing ball Horizontal direction from the top of the towee as shown in fig. find following:

① time

② Range

③ 2sec. velocity speed.

④ α



1 yard = 3 feet.

① $\text{Time} = \sqrt{\frac{2H}{g}}$

$= \sqrt{\frac{2 \times 180}{10}}$

$= \sqrt{36}$

time = 6 sec.

② Range $= u \times t$

$= 60 \times 6$

$= 360\,m$

$60 \times \cancel{2} = \overset{60}{\cancel{120}}\,m$

$g \times t = 20\,m$

③ $v = 120\hat{i} + 20\hat{j}$

$= \sqrt{(60)^2 + (400)}$

$= \sqrt{\underset{3600}{\cancel{14400}} + 400}$

$= \sqrt{4000} = 20\sqrt{10}$

$= \sqrt{} = \cancel{200}$

$\alpha = \frac{20}{60} = +\frac{1}{3}$

(4) Find the speed of striking

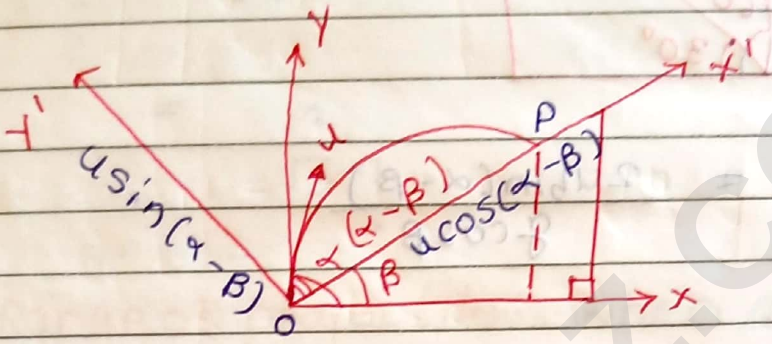
$$u = 60$$
$$gt = 60$$

$$v = 60i + 60j$$
$$= \sqrt{(60)^2 + (60)^2}$$
$$= \sqrt{3600 + 3600}$$
$$= \sqrt{7200}$$
$$= \sqrt{3600 \times 2} = 60\sqrt{2}$$

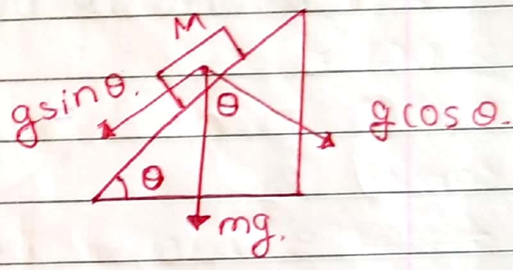
$$\tan\alpha = \frac{60}{60}$$
$$\tan\alpha = 1$$

$$\boxed{\therefore \alpha = 45^\circ}$$

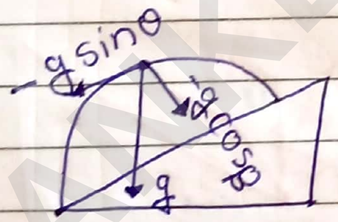
* It a body is projected at an angle α as shown in fig. we can disscus it motion undee.



$$H_{net} = 0 \quad \text{— (1)}$$



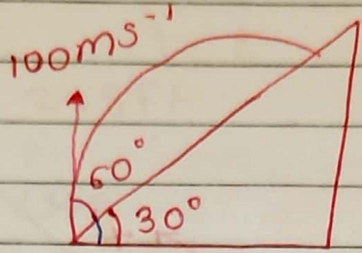
1. Time of Flight [OP].



$$T = \frac{2u_y}{a_y} = \frac{2u(\sin\alpha - \beta)}{g\cos\beta}.$$

For the given projectile find the total time for the following projectile

(i)



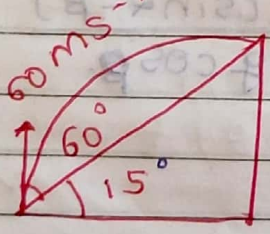
$$T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

$$= \frac{2\times10\cancel{0}\ \sin(60^\circ-30^\circ)}{1\cancel{0}\times\cos30^\circ}$$

$$= \frac{20}{\sqrt3}\times\frac{1}{\cancel{2}}\times\cancel{2}$$

$$T = \frac{20}{\sqrt3}$$

(ii)



$$T = \frac{2u_y}{a_y}$$

$$= \frac{2u\sin(\alpha-\beta)}{g\cos\beta} = \frac{2\times6\cancel{0}\times\sin(60-15)}{1\cancel{0}\times\cos(\cancel{60-45})}$$

45 15°

$\cos(A-B)$

$= \cos A \times \cos\beta + \sin A \times \sin B.$

$= 12 \times \frac{1}{\sqrt{2}}$

$\cos(45-30)$

$= \cos 45^\circ \times \cos 30^\circ + \sin 45^\circ \times \sin 30^\circ$

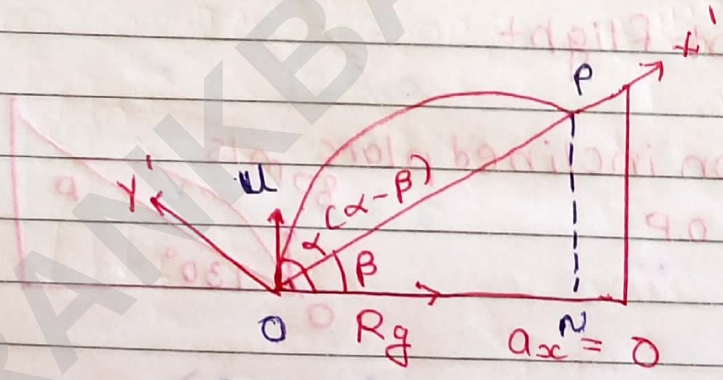
$= \frac{12 \times 1}{\frac{1}{\sqrt{2}}(\sqrt{3}-1)\sqrt{2}}$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)$

$= \frac{1}{\sqrt{2}}(\sqrt{3}+1)$

$= \frac{12}{\sqrt{3}+1}$

\# **Range of projectile:** When a body is thrown over an inclined plane then to calculate its range we can processed as under:



$ON = R_g = (u\cos\alpha) \times T + \frac{1}{2} \times 0 \times T^2$

$= (u\cos\alpha) \times \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$

$= \frac{2u^2\cos\alpha\sin(\alpha-\beta)}{g\cos\beta}$

$OP = ?$

~~$OP = 0$~~

$$\cos\beta = \frac{ON}{OP}$$

$$OP = \frac{ON}{\cos\beta} = \frac{2u^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$$

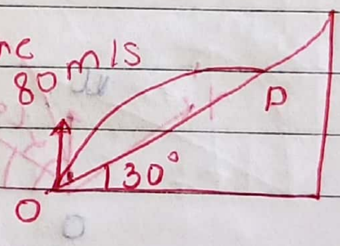
$$\boxed{OP = \frac{2u^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}}$$

For a given projectile motion find the following:

① Time of flight

② R_g.
Range on inclined plane

③ Range OP



$$\text{Time} = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

$$= \frac{2\times 80\times\sin(60-30)}{10\times\cos 30^\circ}$$

$$= \frac{16}{\sqrt{3}}\times\frac{1}{2}\times\cancel{2}$$

$$= \frac{16}{\sqrt{3}}$$

$$Rg = u\cos\alpha \times T$$

$$= \cancel{80}^{40} \times \frac{1}{\cancel{2}} \times \frac{16}{\sqrt{3}}$$

$$= \frac{40 \times 16}{\sqrt{3}}$$

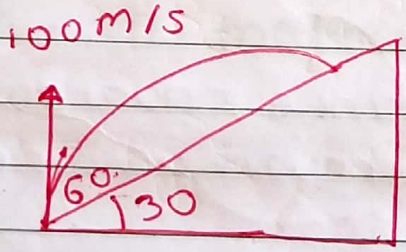
$$= \frac{640}{\sqrt{3}}$$

$$OP = \frac{ON}{\cos\beta}$$

$$= \frac{640 \times 2}{\sqrt{3} \times \sqrt{3}} \qquad \frac{u^2(\sin(2\alpha-\beta) - \sin\beta)}{g\cos^2\beta}$$

$$= \frac{1280}{3}$$

$$= 426.6$$



$$\text{Time} = \frac{2u\sin(\alpha-\beta)}{g\cos\beta} = \frac{2 \times 10\cancel{0}\ \cancel{\sin}\ 60°}{1\cancel{0} \times \sqrt{3}} \frac{\sqrt{3}}{\cancel{2}\ 2}$$

$$= \frac{20}{\cancel{\sqrt{3}}}$$