Ch. SIMPLE HARMONIC MOTION (S.H.M.)

All oscillatory motion are periodic but all periodic motion are not oscillatory.

Oscillatory / Harmonic Motion – To & fro

$F = -kx^n \quad n = 1, 3, 5, 7, 9 \ldots$ (odd no.)

Simple Harmonic Motion -

$F = -kx^1 \quad (n = 1)$

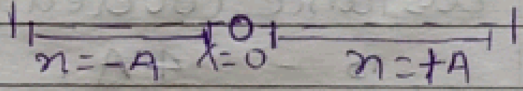
when x is +ve → $F = -kx$

when x is -ve - $F = kx$

direction of force & accⁿ. is always towards mean

Understanding S.H.M & basic terms related to S.H.M

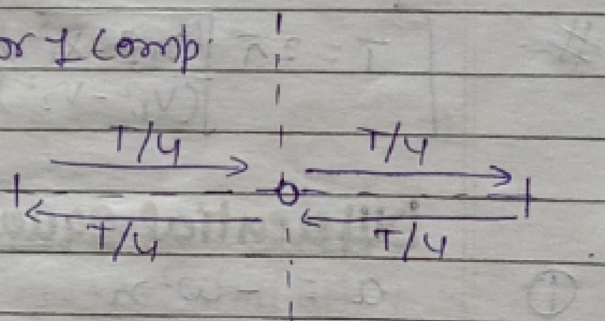
① Amplitude - A



② Time period (T) - Time taken for 1 comp. oscillation.

$t = T$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$



③ Frequency - no. of oscillation in 1 sec.

$f = \frac{1}{T}$

angular frequency (ω) $= \frac{2\pi}{T} = 2\pi f$

④ Restoring Force (F) $\quad F = -kx \quad\quad k = m\omega^2$

$[\ F = -m\omega^2 x\ ]$

(5) Acceleration (a) - always acts towards mean.

$a = \frac{F}{m} = -\frac{m\omega^2 x}{m} = -\omega^2 x$

$[a = -\omega^2 x]$

$|a_{max}| = \omega^2 A$

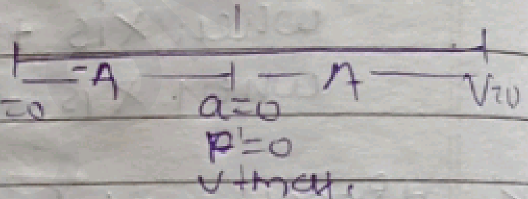
a_{max} - at extreme position

a_{min} - at mean position

(6) Velocity (V) -

$V = \pm \omega^2 (A^2 - x^2)$

$V = \pm \omega \sqrt{A^2 - x^2}$



$[\ |V_{max}| = \omega A\ ]$

\# distance covered by a particle undergoing SHM is - 4A.

\# $\left[T = 2\pi \sqrt{\frac{(x_2^2 - x_1^2)}{(v_1^2 - v_2^2)}} \right]$ standard eq.

Differential equation of S.H.M

(1) $a = -\omega^2 x$ — (i)

If disp. - x $\quad v = \frac{dx}{dt}$

$a = \frac{dv}{dt}$, $\quad a = \frac{d^2x}{dt^2}$ — (ii)

from (i) & (ii) $\frac{d^2x}{dt^2} = -\omega^2 x$

$\left[\text{differential eq. of SHM} = \frac{d^2x}{dt^2} + \omega^2 x = 0 \right]$

Equation of S.H.M

Dt
Pg
B+

Let's consider S.H.M along x axis.

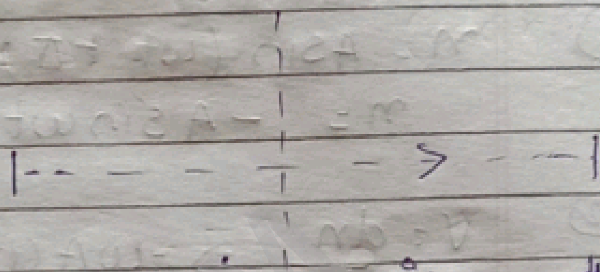
① $x = A\sin(\omega t + \phi)$ $\qquad \phi$ – initial phase

② $x = A\cos(\omega t + \phi)$ $\qquad [\omega t + \phi]$ – phase

*) Case - 1

i) eq. of displacement.

$x = A\sin(\omega t + \phi)$



at $x = 0$ $t = 0$

$0 = A\sin(0 + \phi)$ $\qquad x = 0$ initial condt.

$0 = A\sin\phi \quad \sin\phi = 0$

$\phi = 0^\circ$ or 180°

ii) eq. of velocity (v)

$v = \frac{dx}{dt} \Rightarrow x = A\sin(\omega t + \phi)$

$v = \omega A\cos(\omega t + \phi)$

at $t = 0$

$v = \omega A\cos\phi \qquad \cos\phi = +1 \quad \phi = 0^\circ$

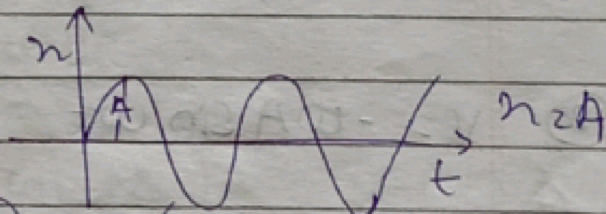
$[\, x = A\sin(\omega t) \,]$

$v = \frac{dx}{dt} = \omega A\cos(\omega t)$

(ii) eq. of accen. – (a) $\quad a = \frac{dv}{dt} = -\omega^2 A\sin(\omega t)$.

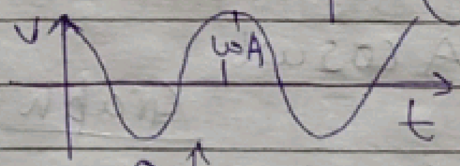
Graphs

① $x = A\sin\omega t$



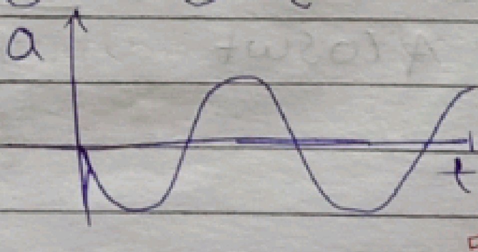
② $v = \omega A\cos\omega t$

$v_{max} = \omega A$



③ $a = -\omega^2 A\sin\omega t$

$a_{max} = \omega^2 A$

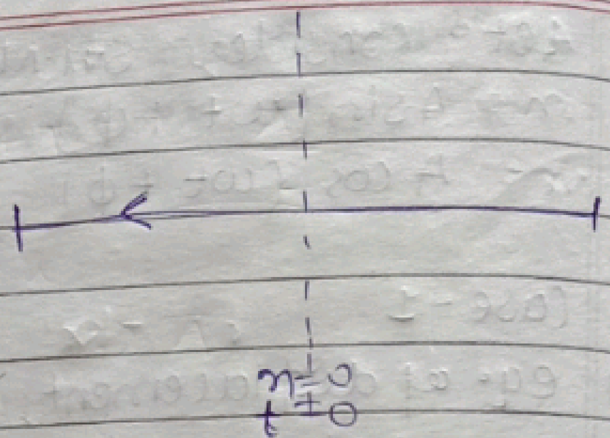


JOY

Case - II

$x = A\sin(\omega t + \phi)$

$\phi = 180^\circ$ or π



① $x = A\sin(\omega t + \pi)$

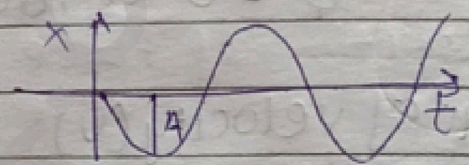
$x = -A\sin\omega t$

② $v = \frac{dx}{dt} \Rightarrow -\omega A\cos\omega t$

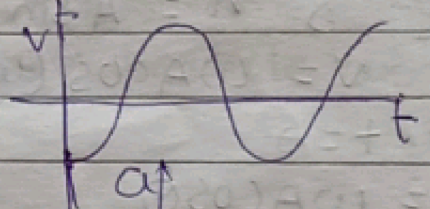
③ $a = \frac{dv}{dt}$ or $-\omega^2 x \Rightarrow \omega^2 A\sin\omega t$

Graph

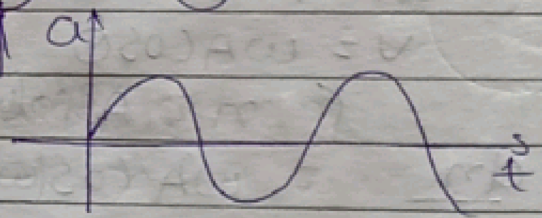
① $x = -A\sin\omega t$



② $v = -\omega A\cos\omega t$

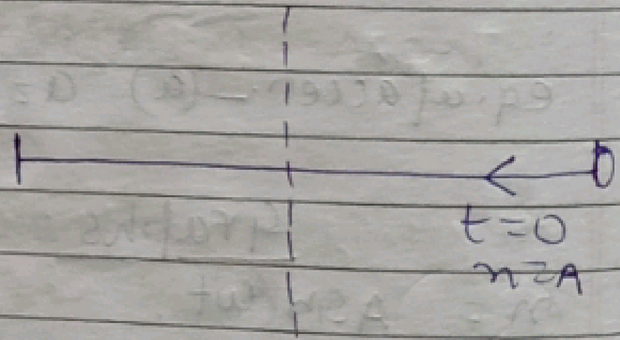


③ $a = \omega^2 A\sin\omega t$



Case III

① $x = A\cos\omega t$

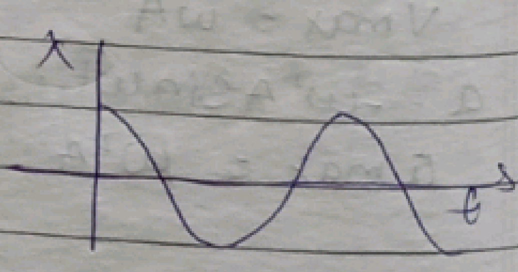


② $v = -\omega A\sin\omega t$

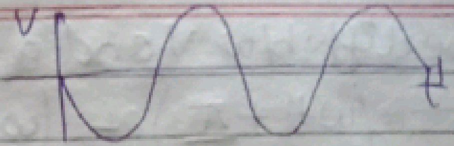
③ $a = -\omega^2 A\cos\omega t$

Graph

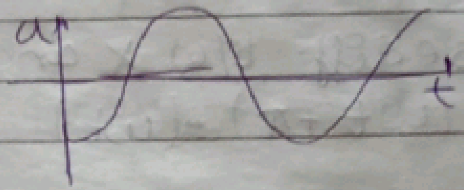
① $x = A\cos\omega t$



(2) $v = -\omega A \sin \omega t$

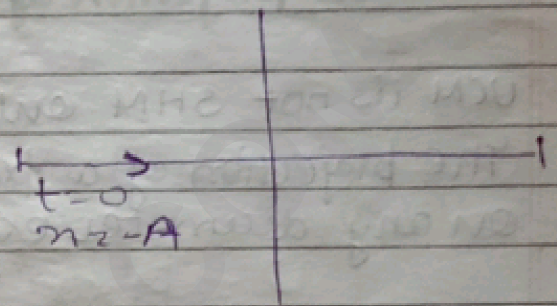


(3) $a = -\omega^2 A \cos \omega t$



Case-IV

(1) $x = -A \cos \omega t$

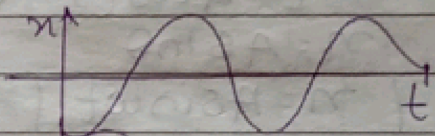


(2) $v = \omega A \sin \omega t$

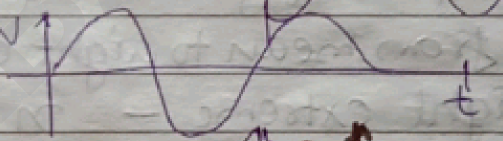
(3) $a = \omega^2 A \cos \omega t$

Graph

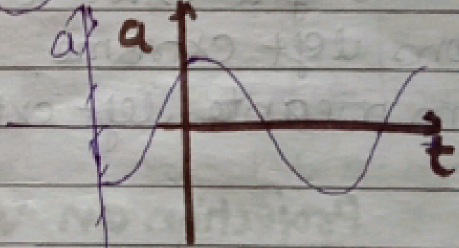
(1) $x = -A \cos \omega t$



(2) $v = \omega A \sin \omega t$



(3) $a = +\omega^2 A \cos \omega t$



Phase & Phase difference

$x = A \sin(\omega t)$ $[\omega t]$ — Phase

$v = \omega A \cos(\omega t) \Rightarrow \omega A \sin\left(\omega t + \frac{\pi}{2}\right)$ $\Rightarrow \sin(90 + \theta) = \cos\theta$

$a = -\omega^2 A \sin(\omega t) \Rightarrow \omega^2 A \sin(\omega t + \pi)$ $\sin(180 + \theta) = -\sin\theta$

Phase diff. b/w x and v

$\omega t - \omega t + \frac{\pi}{2} \Rightarrow \frac{\pi}{2}$

Phase diff. b/w v and a

$$\left(\omega t+\frac{\pi}{2}\right)-(\omega t+\pi)=\frac{\pi}{2}$$

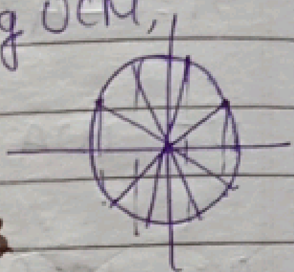
$\cos 270 = 0$
$\cos 90 = 0$
$\cos 180 = -1$
$\cos 360 = -1$

Phase diff. b/w x and a

$$(\omega t+\pi)-(\omega t) \Rightarrow \pi$$

S.H.M. as projection of uniform circular motion

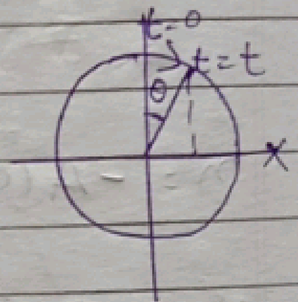
⇒ UCM is not SHM only periodic.
⇒ The projection of a particle undergoing UCM, on any diameter executes SHM.



Projection SHM on horizontal diameter

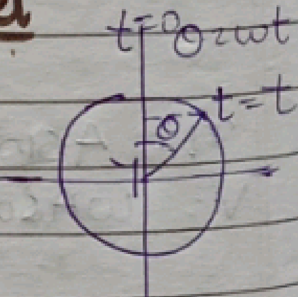
$\theta=\omega t$
$x=A\cos(90-\theta)$
$x=A\sin\theta$

① $[x=A\sin\omega t]$
from mean to right extreme.

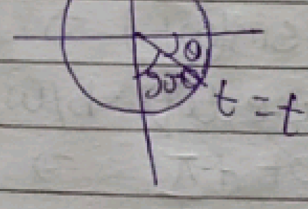


② From right extreme – $x=A\cos\omega t$
③ From left extreme . $x=-A\cos\omega t$
③ From mean to left extreme - $x=-A\sin\omega t$

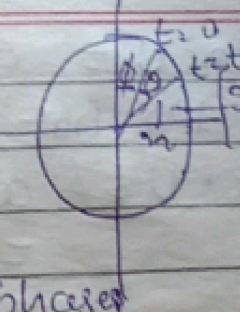
Projection on vertical diameter



① $y=A\cos\theta \Rightarrow A\cos\omega t$
② $y=A\cos(90-\theta)=-A\sin\theta$
$y=-A\sin\omega t$



When initial phase is given



$x = A\cos(90 - \theta + \phi)$

$x = A\sin(\omega t + \phi)$

ϕ - initial phase.

$x = A\sin(\omega t + \phi)$

Energy in S.H.M

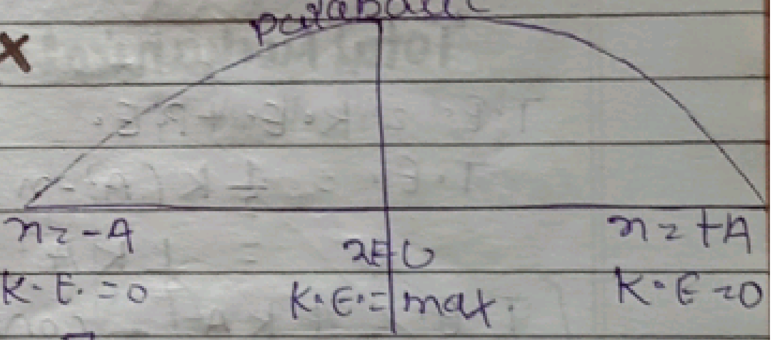
Kinetic Energy –

- $K.E = \frac{1}{2}mv^2 \Rightarrow \frac{1}{2}m\omega^2(A^2 - x^2)$ $(m\omega^2 = k)$

$$\boxed{K.E. = \frac{1}{2}k(A^2 - x^2)}$$

- Min. K.E. = at $x = A$ $K.E. = 0$ at extremem.
- Max. K.E = at $x = 0$ $\left[K.E. = \frac{1}{2}kA^2\right]$ – at mean.

Graph K.E. V/S ~~x~~ x



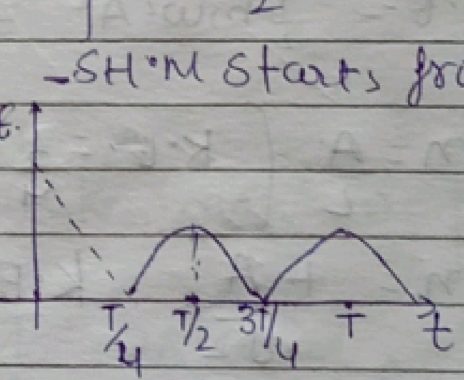
$K.E. = \frac{1}{2}mv^2$

$x = A\sin\omega t$, $v = \omega A\cos\omega t$

$$\left[K.E. = \frac{1}{2}m\omega^2A^2\cos^2\omega t\right] \Rightarrow \frac{1}{2}kA^2\cos^2\omega t$$

$\Rightarrow K.E. = K.E_{max}\cos^2\omega t$ – S.H.M starts from mean posit

\# K.E. – periodic repeats itsels after $\frac{T}{2}$



Potential Energy

① $\left[U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2\right]$ – (same as spring force, $F = -kx$)

Dt
Pg
B+

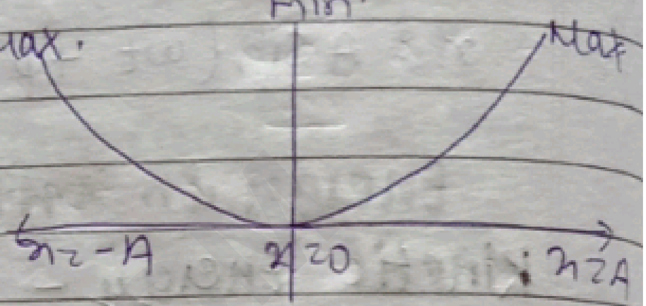
- min. U = at $x=0$ $U=0$ — at mean

It's not always 0 if it has some initial U.

- Max. U = at extreme $x=A$

$= \frac{1}{2}KA^2 \Rightarrow \frac{1}{2}m\omega^2A^2$

Graph U v/s x

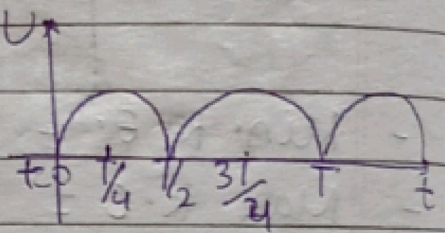


$U = \frac{1}{2}kx^2$ $\quad x = A\sin\omega t$

$\left[U = \frac{1}{2}KA^2\sin^2\omega t\right] \Rightarrow \frac{1}{2}m\omega^2A^2\sin^2\omega t$

$\left[U = U_{max}\sin^2\omega t\right]$

U is periodic with Timeperiod $T/2$



Total Mechanical energy in SHM

$T.E. = K.E. + P.E.$

$T.E. = \frac{1}{2}K(A^2-x)^2 + \frac{1}{2}kx^2$

$= \frac{1}{2}KA^2 - \cancel{\frac{1}{2}kx^2} + \cancel{\frac{1}{2}kx^2}$

$\left[T.E. = \frac{1}{2}KA^2\right]$ — constant it not vary with t & x.

$T.E. = \frac{1}{2}m\omega^2A^2 = K.E._{max} = U_{max}$.

At $x = \frac{A}{2}$ $\left[K.E. = \frac{3}{4},\ U = \frac{1}{4} \text{ of } T.E\right]$

at $x = \pm\frac{A}{\sqrt{2}}$ $\quad K.E. = P.E.$

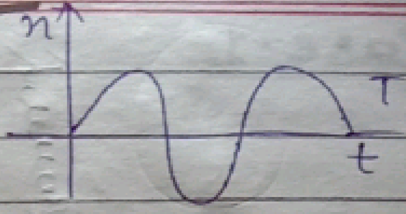
In comp. SHM $4U = K.E.$ during one comp. period.

Periodicity of x, v, a, K.E., P.E, T.E.

(1) $x = A\sin\omega t$

Periodic - SHM

$T = \frac{2\pi}{\omega}$ ω - Frequency

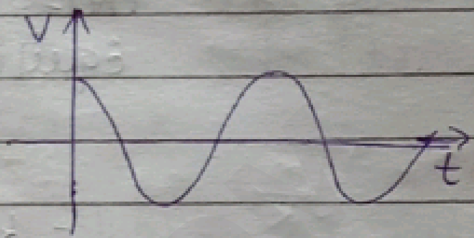


$T = \frac{2\pi}{\omega}$

(2) $V = \omega A\cos\omega t$

Periodic - SHM

ω - frequency.

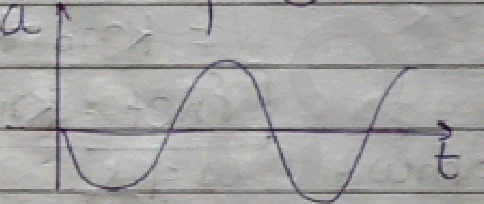


$T = \frac{2\pi}{\omega}$

(3) $a = -\omega^2 A\sin\omega t$

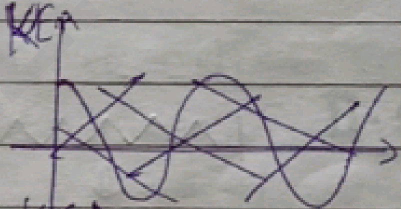
Periodic - SHM

ω - frequency.



$T = \frac{2\pi}{\omega}$

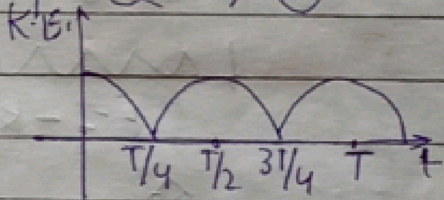
(4) $K.E. = (K.E.)_{max}\cos^2\omega t$



Time period $= \frac{T}{2}$ of SHM

Periodic but not SHM

frequency - 2ω of SHM

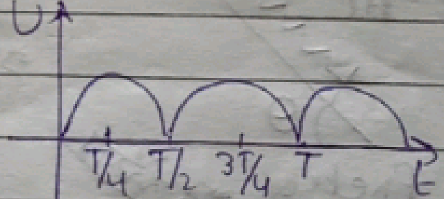


(5) $P.E. = (P.E)_{max}\sin^2\omega t$

Time period - $\frac{T}{2}$ of SHM

Periodic

Frequency - 2ω of SHM



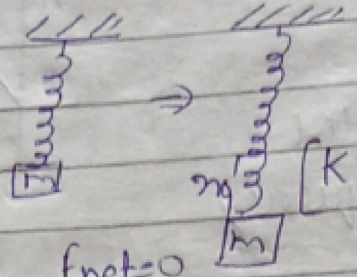
Time period of any SHM

$f = -kx = -m\omega^2 x$ $\quad K = m\omega^2 \quad \omega = \sqrt{\frac{K}{m}}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{K}{m}}} \Rightarrow \left[T = 2\pi\sqrt{\frac{m}{K}}\right]$ Always.

Spring Mass System

Case-1

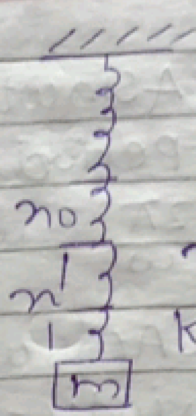


$[Kx_0 = mg]$ x_0

$x' = x_0 + x$

$K(x_0+x) > mg$

Fnet=0
Equilibrium



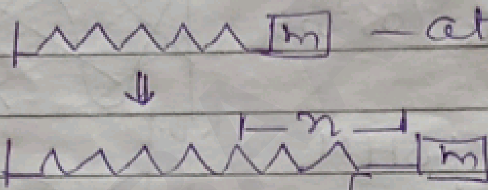
$F_{net} = K(x_0+x) - mg$

$= Kx_0 + Kx - mg \Rightarrow mg + Kx - mg$

$\boxed{F_{net} = Kx}$

$K = m\omega^2$ $\quad \omega = \sqrt{\frac{K}{m}}$ $\quad T = \frac{2\pi}{\omega} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{K}}}$

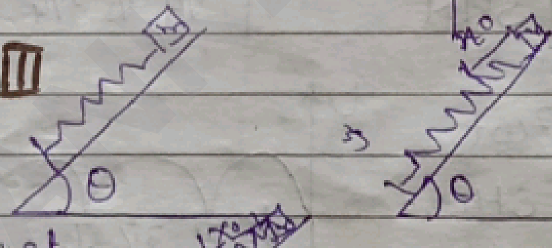
Case-II



— at eq. Fnet=0

$F = -Kx$ $\quad \boxed{T = 2\pi\sqrt{\frac{m}{K}}}$

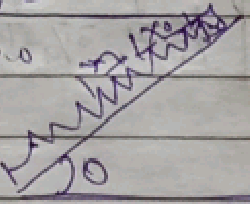
Case-III



$Kx_0 = mg\sin\theta$

Fnet $\neq$ 0

Fnet=0

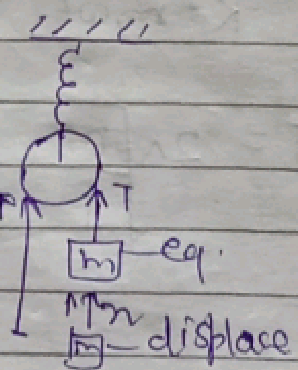


$mg\sin\theta = K(x_0+x)$

$mg\sin\theta = mg\sin\theta + Kx$

$F_{net} = -Kx$ $\quad \boxed{T = 2\pi\sqrt{\frac{m}{K}}}$

Case-IV



$\Rightarrow F = \frac{-Kx}{2}$ $\quad 2T = F$

$T = \frac{Kx}{4}$ $\quad F = -\frac{K}{4}x$

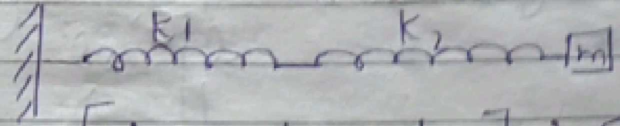
$T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow 2\pi\sqrt{\frac{m}{\frac{K}{4}}}$

$\boxed{T = 4\pi\sqrt{\frac{m}{K}}}$

Combination of Spring

B+

① **series combination**



$$\left[\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}\right] \text{ or } \left[K_{eq} = \frac{K_1 K_2}{K_1 + K_2}\right] \text{ for 2 spring}$$

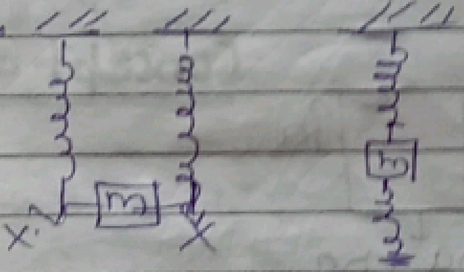
spcl case - If $K_1 = K_2 \Rightarrow K_{eq} = \frac{K}{2}$

In Series 1 end on each spring always connect with each other

② **Parallel combination**

Always one end is fixed & other end is connected with mass.

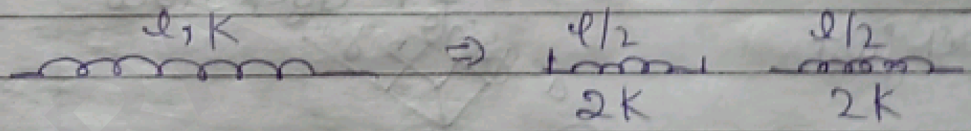
$$[K_{eq} = K_1 + K_2]$$



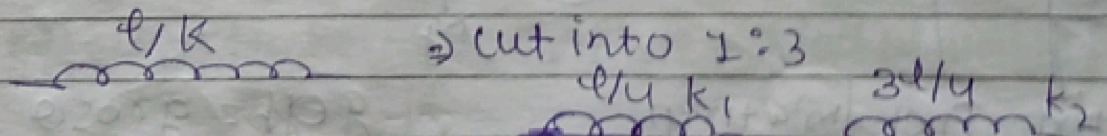
Cutting of Spring

- Spring cons. $K \propto \frac{1}{l}$

①

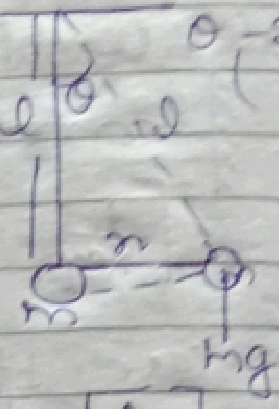


②



$K_1 = 4K$ $\quad$ $K_2 = \frac{4}{3}K$

Simple Pendulum



θ - small ($5^\circ - 10^\circ$)

Restoring Force $= mg\sin\theta$

$\sin\theta = \frac{x}{l}$

$F = -\frac{mgx}{l}$ $\quad F = \frac{mgx}{l}$

$\left(\frac{mg}{l} = k\right)$

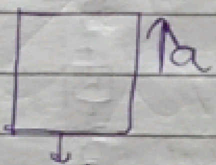
$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow 2\pi\sqrt{\frac{m}{\frac{mg}{l}}} \Rightarrow \left[2\pi\sqrt{\frac{l}{g}}\right]$

- only for small displacement
- independent of mass of bob.

\# $n_1 t_1 = n_2 t_2$ $\quad$ n - no. of oscillation.

Concept of $g_{effective}$

(1)



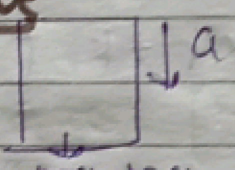
$ma + mg$

$m(a+g)$

$T = 2\pi\sqrt{\frac{l}{g+a}}$

T↓ → clock fast

(2)

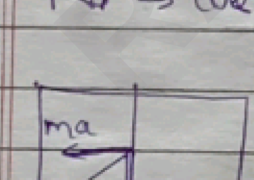


$mg - ma$ $\quad g_{eff} = g - a$

$T = 2\pi\sqrt{\frac{l}{g-a}}$

T↑ clock slow.

(3)



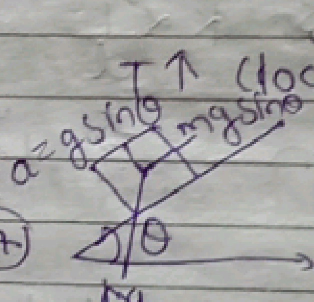
Fnet $\quad$ ma $\quad F_{net} = \sqrt{m^2a^2 + m^2g^2}$

$g_{eff} = \sqrt{a^2 + g^2}$

$\left[T = 2\pi\sqrt{\frac{l}{a^2+g^2}}\right]$

- T↓ - clock fast

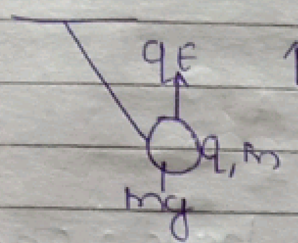
(4)



$g_{eff} = g\cos\theta$

$\left[T = 2\pi\sqrt{\frac{l}{g\cos\theta}}\right]$

Oscillation of a simple pendulum in a Electric Field B+

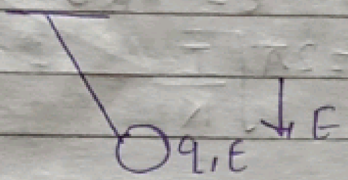


net force = mg − qE

$a = \frac{mg - qE}{m}$

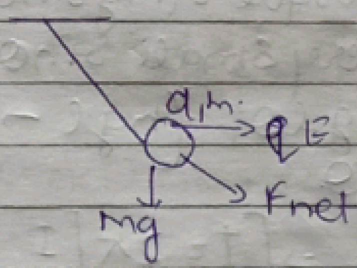
$$\boxed{g_{eff} = g - \frac{qE}{m}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}}$$ T↑ − Clock slow.



$\left(g_{eff.} = g + \frac{qE}{m}\right)$

$F_{net} = qE + mg$ $$\boxed{T = 2\pi\sqrt{\frac{l}{g + \frac{qE}{m}}}}$$ T↓ → Clock Fast.



$F_{net} = \sqrt{m^2g^2 + q^2E^2}$

$\cancel{m}g_{eff} = \cancel{m}\sqrt{g^2 + \frac{q^2E^2}{m^2}}$

$g_{eff} = \sqrt{g^2 + \frac{q^2E^2}{m^2}}$

$$\boxed{T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{q^2E^2}{m^2}}}}}$$ T↓ → Clock fast

Effect of temperature on time period of simple pendulum

$T = 2\pi\sqrt{\frac{l}{g}}$

On ↑ Temp. → l↑ → T↑ − Clock slow time lose

On ↓ Temp. − l↓ → T↓ − Clock fast time gain

On changing temp. by $\Delta\theta$ T changes by ΔT

$$\boxed{\Delta T = \frac{1}{2}\alpha T \Delta\theta}$$

ΔT — Change in time period.

α — Coefficient of linear expansion

T — time period.

$\Delta\theta$ — change in temp.

loss ~~of~~ or gain in time t

$$\boxed{\Delta t = \frac{\Delta T}{T} \times t}$$

gain or loss in time t

Second Pendulum → pendulum have 2 sec. time period.

Angular S.H.M

Linear S.H.M.

$F = -Kx$

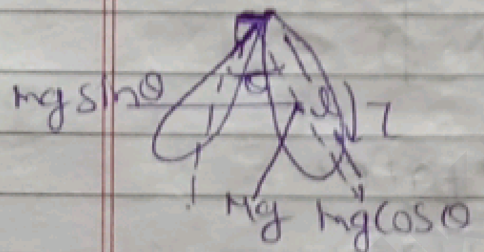
$T = 2\pi\sqrt{\frac{m}{K}}$

Angular S.H.M

$\tau = -K\theta$

$T = 2\pi\sqrt{\frac{I}{K}}$ I - moment of Inertia

Physical Pendulum (compound pendulum)



restoring Torque = (mg sinθ) × l

θ - very small sinθ → θ

$\tau = -mgl\theta$ ($mgl \to K$)

$\tau = K\theta$

$T = 2\pi\sqrt{\frac{I}{K}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{I}{mgl}}}$

I - M.O.I about pivoted point.

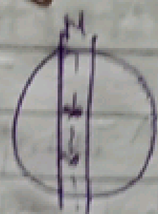
l - dis. b/w pivoted point & centre of gravity l.

S.H.M of body in a tunnel along any chord (Including diameter) of earth.

(1) Along diameter

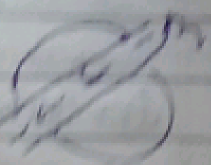
- M perform SHM

- mean position → centre of earth.



$T = 2\pi\sqrt{\frac{R_e}{g}} \Rightarrow 84.6$ min

(ii) Any other chord
- M perform SHM about centre of chord



$T = 2\pi\sqrt{\frac{R_0}{g}}$ ⇒ 84.5 min

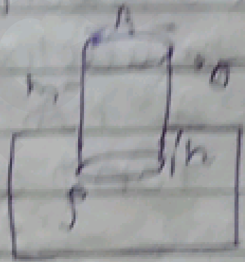
To reach 1 side to other side - $t = 42.3$

for 1 comp round it take - 84.6

Oscillation of floating body

⇒ If body is slightly displaced from mean position, body perform SHM

(1) $\boxed{T = 2\pi\sqrt{\frac{m}{A\rho g}}}$ A - Area of object, m - mass of object, ρ - density of fluid.



(2) $\boxed{T = 2\pi\sqrt{\frac{h}{g}}}$

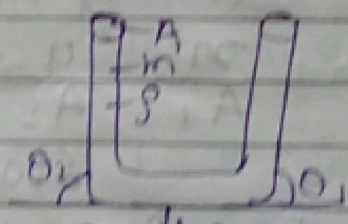
⇒ condition for floatation ⇒ $\rho_f l_f = \rho_s l_s$
$= \rho \times h = \sigma L$

In terms of density

$\boxed{T = 2\pi\sqrt{\frac{l\sigma}{\rho g}}}$

Oscillation of liquid column

(1) $T = 2\pi\sqrt{\frac{m}{A\rho g(\sin\theta_1 + \sin\theta_2)}}$



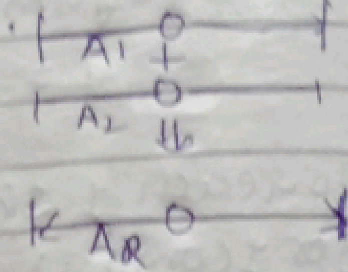
A - Area of cross section of tube
ρ - density of liquid
m - mass of liquid in both column

In this case $\theta_1 = \theta_2 = 90°$

$T = 2\pi\sqrt{\frac{m}{A\rho g(\sin 90 + \sin 90)}}$

$\boxed{T = 2\pi\sqrt{\frac{m}{2A\rho g}}}$

Combination of two or more SHM



① In same direction

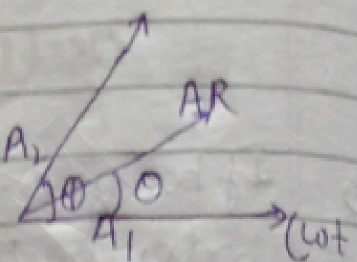
$x_1 = A_1 \sin \omega t$

$x_2 = A_2 \sin(\omega t + \phi)$

$x_R = x_1 + x_2$

$x_R = A_1 \sin \omega t + A_2 \sin(\omega t + \phi)$

By Vector / Phasor method



$x_R = A_1 \sin \omega t + A_2 \sin(\omega t + \phi)$

$[x_R = A_R \sin(\omega t + \phi)]$

By parallelogram law,

$A_R^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos \phi$

$\tan \phi = \dfrac{A_2 \sin \phi}{A_1 + A_2 \cos \phi}$

\# Express x_1 & x_2 in $\sin \omega t$ & +ve.

\# If $x = A \sin \omega t + B \cos \omega t$

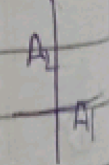
$A_R = \sqrt{A^2 + B^2}$

\# $\left[\begin{array}{l} \cos \omega t = \sin(90 + \omega t) \\ -\cos \omega t = \sin(90 - \omega t) \end{array}\right]$

② In two perpendicular direction.

⇒ Case - I $\quad x = A_1 \sin \omega t \Rightarrow \sin \omega t = \dfrac{x}{A_1}$

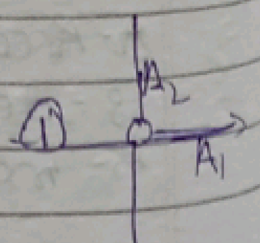
$y = A_2 \sin \omega t \Rightarrow \sin \omega t = \dfrac{y}{A_2}$



$\dfrac{x}{A_1} = \dfrac{y}{A_2} \qquad y = \dfrac{A_2}{A_1} x$

Slope → Straight line.

⇒ Case - II ⇒ ① $x = A_1 \sin \omega t \quad \sin \omega t = \dfrac{x}{A_1}$ — ①



$y = A_2 \cos \omega t \Rightarrow A_2 \sin(\omega t + \pi)$

$\cos \omega t = \dfrac{y}{A_2}$ — ⓑ

$$\sin^2\omega t + \cos^2\omega t = \frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1$$

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1 \quad \text{Ellipse}$$

$\Rightarrow$ Case III – $x = A\sin\omega t$

$y = A\cos\omega t$

$\sin\omega t = \frac{x}{A} \qquad \cos\omega t = \frac{y}{A}$

$\sin^2\omega t + \cos^2\omega t = 1 - \frac{x^2}{A^2} + \frac{y^2}{A^2} = 1$

$x^2 + y^2 = A^2 \Rightarrow$ circle, radius – A

centr(0,0)

$\Rightarrow$ Case IV – $x = A_1\sin\omega t \qquad \sin\omega t = \frac{x}{A_1}$

$y = -A_2\sin\omega t \qquad \sin\omega t = \frac{-y}{A_2}$

$\frac{x}{A_1} = \frac{-y}{A_2} \qquad y = \frac{-A_2}{A_1}x$

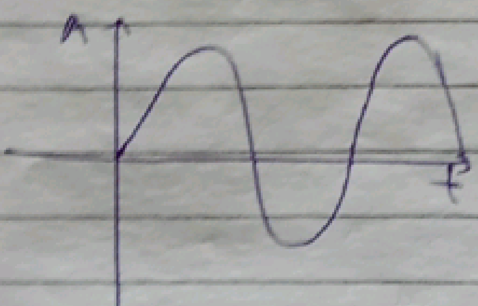
– Straight line –ve slope.

Free Oscillation

- Natural Vibration / oscillation
- No ext Force is present (viscous, Friction)
- Oscillation will continue Forever
- No energy is wasted.
- Amplitude will remain same Forever.
- Frequency of free oscillation = Natural frequency

$\downarrow$

F_0 or ω_0



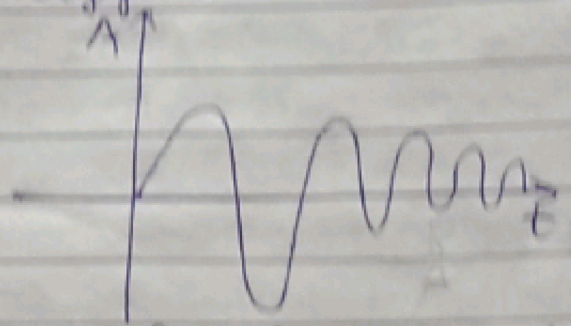
$\omega_0 = \sqrt{\frac{K}{m}}$

$K = m\omega_0^2$

Damped oscillation

1. Oscillation in presence of ext. force, will not continue forever

- Amplitude will ↓ with time
- energy will be wasted.



- Damping force = $-bV$ b - damping cons
  V - Velocity of object
- Amplitude with time. $A = A_0 e^{-\gamma t}$ $\gamma = \frac{b}{2m}$
- Frequency of damped oscillation is always less than natural frequency.

Natural	Damped
$\omega_0 = \sqrt{\frac{K}{m}}$	$\omega = \sqrt{\omega_0^2 - \gamma^2}$
$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$	$F_d = -bV$

$\gamma = \frac{b}{2m}$

- Equation of damped oscillation.

$$m\frac{d^2x}{dt^2} + \omega\frac{dx}{dt} + bx = 0$$

$$ma + \omega V + bx = 0$$