31/10/19 L-1 TL-5

CENTRE OF MASS

* It is the point located inside or outside body where the whole mass of body is supposed to be concentrated.

* It is the point about which the sum of all the ~~mass movements~~ Mass moment = 0

* <u>center of Gravity</u> :- It is the point where weight (mg) of the body acts

usually COM & COG are considered to be same.

- For buildings (high) COG & COM will be diff.

1/10/19

L-2

TL-59

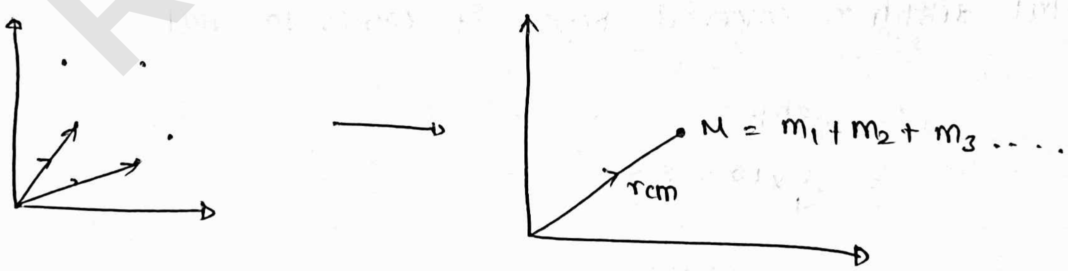
* <u>Mass of ~~Movements~~ Moment</u>

- It is a vector quantity. which is given by product of mass of particle and its position vector w.r.t origin.

- Mass Moment OR Moment of Mass $= m\vec{r}$ (Kg-m)

Moment arm $\vec{r}$

* <u>system of particles</u>



$$m_1\vec{r_1} + m_2\vec{r_2} + \ldots\ldots + m_n\vec{r_n} = M\vec{r_{cm}}$$

$$\boxed{\vec{r}_{cm} = \frac{m_1\vec{r_1} + m_2\vec{r_2} \ldots + m_n\vec{r_n}}{m_1 + m_2 \ldots + m_n} = \frac{\Sigma(m\vec{r})}{\Sigma(m)}}$$

* if COM is at origin $r_{com} = 0 \Rightarrow \Sigma(m\vec{r}) = 0$.

$$\boxed{\vec{r}_{cm} = \frac{\int m\vec{r}}{\int m}}$$

① **co-ordinate of COM**
(Position)

$$x_{cm} = \frac{\int dm \cdot x}{\int dm}$$

$$y_{cm} = \frac{\int dm \cdot y}{\int dm}$$

$$z_{cm} = \frac{\int dm \cdot z}{\int dm}$$

② **velocity of COM**

$$\vec{v}_{cm} = \frac{d(\vec{r}_{cm})}{dt}$$

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \dots + m_n\vec{v}_n}{m_1 + m_2 \dots + m_n}$$

$$\boxed{\vec{v}_{cm} = \frac{\int M \cdot \vec{v}}{\int m} = \frac{\vec{P}_{syst.}}{\int m}}$$

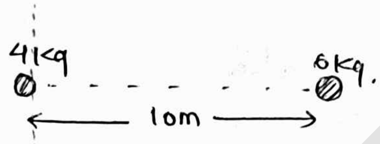
③ **Acc of COM**

$$\vec{a}_{cm} = \frac{d(\vec{v}_{cm})}{dt}$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + \dots + m_n\vec{a}_n}{m_1 + \dots + m_n}$$

$$\boxed{\vec{a}_{cm} = \frac{\int m \cdot \vec{a}}{\int m} = \frac{\vec{F}_{ext}}{\int m}}$$

Que.



locate COM

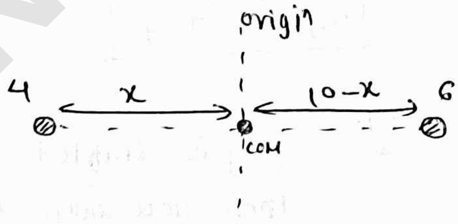
sol.

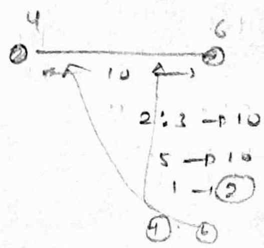
$$x_{cm} = \frac{4(0) + 6(10)}{10} = 6m$$

$$y_{cm} = \frac{4(0) + 6(0)}{10} = 0$$

→ COM is at dist. 6m from 4kg on the line joining the particle

Method-2



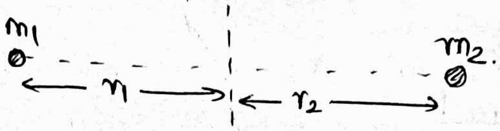


At origin COM = 0

$$-(4x) + 6(10-x) = 0$$

$$10x = 60$$

$$\boxed{x = 6m}$$



$$r_1 + r_2 = r$$

$$m_1(-r_1) + m_2 r_2 = 0$$

$$\boxed{m_1 r_1 = m_2 r_2}$$

$\frac{m_1}{m_2} + 1 = \frac{r_2}{r_1} + 1$

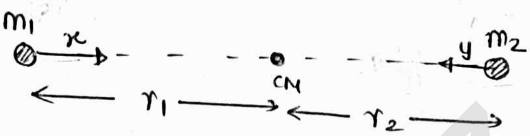
$\frac{m_1 + m_2}{m_2} = \frac{(r_1 + r_2)}{r_1}$

$r_1 = m_2 r$

① $\boxed{\frac{m_1}{m_2} = \frac{r_2}{r_1}}$ ② $\boxed{r \propto \frac{1}{m}}$

$$\boxed{r_1 = \left(\frac{m_2}{m_1 + m_2}\right) r} \qquad \boxed{r_2 = \left(\frac{m_1}{m_1 + m_2}\right) r}$$

Ques #



- where to shift

  If m_1 toward left, m_2 toward Right

  m_1 toward Right, m_2 toward left

- How much

  Initially $m_1 r_1 = m_2 r_2$

  After shifting $m_1(r_1 - x) = m_2(r_2 - y)$

$$\boxed{m_1 x = m_2 y}$$

Ques

3kg is shifted towards CM by 15cm then How much + where the mass 5kg is being shifted to make COM unchanged

sol.

where → Towards ~~Righ~~ CM.

where $3 \times 15 = 5 \times x$

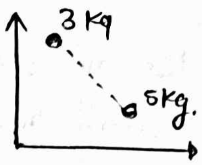
$x = 9\text{ cm}$ right

$$M_1 x = M_2 y.$$

$$3 \times 15 = 5 \times y.$$

$$\boxed{y = 9\text{cm}}$$

Ques.



3 kg is shifted away from CM by 15cm on the line joining the particles then req. shift in 5 kg so that COM will be at same positⁿ.

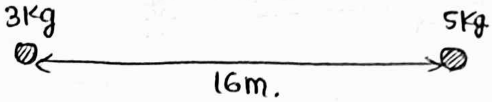
Sol.

Where → Away from CM on LJP.

$\Rightarrow \quad 3 \times 15 = 5 \times y.$

$\boxed{y = 9\text{cm}}$

Ques.



if particles interchange their posi. then find shift in COM.

Sol.

$3x = 5(16-x)$

$3x = 80 - 5x$

$\boxed{x_1 = 10}$

$5x = 3(16-x)$

$5x = 48 - 3x$

$8x = 48$

$\boxed{x_2 = 6}$

$x_1 - x_2 = \underline{4\text{cm}}$ towards 5 kg.

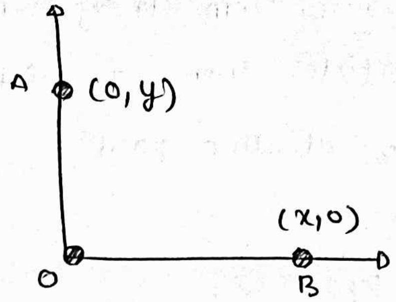
3 : 5 → 16
6 → 16
1 → 2
6 → 10

$M_1 x_1$
$3 \times x = 5(16-x)$
$3x = 80 - 5x$
$8x =$
$x = 10$

10 : 4

Heavier वाले की तरफ shift होगा।

Ques.



Three particles of equal mass are kept as shown find the positⁿ of COM.

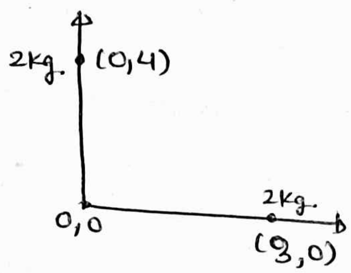
Sol.

$$x_{cm} = \frac{m(0) + m(x) + m(0)}{3m} = \frac{x}{3}$$

$$y_{cm} = \frac{m(0) + m(0) + m(y)}{3m} = \frac{y}{3}$$

$CM\left(\frac{x}{3}, \frac{y}{3}\right)$

Ques.



Where should we put the 3rd particle of equal mass (2kg) so that COM will be at origin.

Sol.

(x, y)

$$x_{cm} = \frac{2(0) + 2(3) + 2x}{6} = 0$$

$x = -3$

$$y_{cm} = \frac{2(4) + 2(0) + 2y}{6}$$

$y = -4$

$(-3, -4)$

$x = \frac{3 \times 8 + 8 \times 0}{4}$

x

$0 = \frac{m_1x_1 + m_2y_2 + m_3x_3}{}$

0

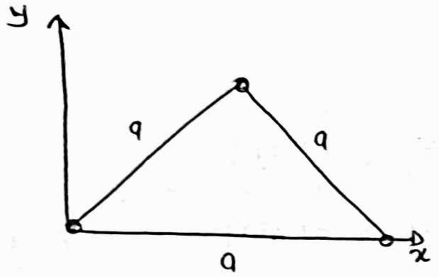
$2 \times 3 + 8$

$6 + 8 + 2x = 0$

14

$(-7, 0)$

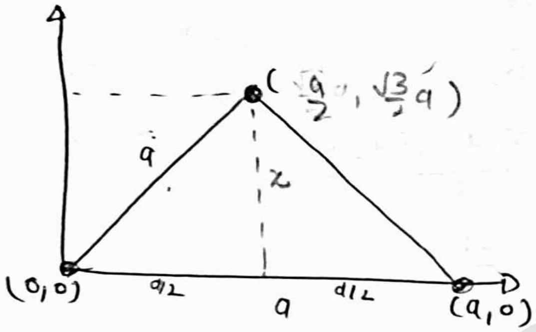
Ques.



3 particle of equal masses are located on vertices of a equilateral Δ, such that one vertice concide with origin

sol.

गलती :- coordinates find करने मे



$a^2 = x^2 + \frac{a^2}{4}$

$x^2 = a^2 - \frac{a^2}{4}$

$x = \frac{\sqrt{3}a}{2}$

$$x_{cm} = \frac{m(0) + m(a) + m\left(\frac{a}{2}\right)}{3m} = \frac{\frac{3ma}{2}}{3m} = \frac{3}{2}ma \times \frac{1}{3m} = a/2$$

$$y_{cm} = \frac{m(0) + m(0) + m\frac{\sqrt{3}}{2}a}{3m} = \frac{\sqrt{3}am}{2} \times \frac{1}{3m} = \frac{a}{2\sqrt{3}}$$

$\Rightarrow$ symmetric distribution of Mass.

C.M will.

Ques. Two particle of masses $m_1 = 1kg$, $m_2 = 2kg$ moving with vel $v_1 = 2\hat{i} + 3\hat{j}$, $v_2 = 3\hat{i} + 2\hat{j}$

find Trajectory of COM?

(a) straight line (b) circle

(c) Parabola (d) Ellipse

sol.

$$\vec{V}_{cm} = \frac{2\hat{i} + 3\hat{j} + 6\hat{i} + 4\hat{j}}{3}$$

$$\vec{V}_{cm} \Rightarrow \frac{8\hat{i} + 7\hat{j}}{3}$$

$$\vec{V}_{cm} \Rightarrow \frac{8\hat{i}}{3} + \frac{7\hat{j}}{3}$$

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

Instaneous dirc^n $\Rightarrow$ $\tan^{-1}\left(\frac{7}{8}\right)$

$\hookrightarrow$ it is time independent means it will not change its dirc^n

Ques. Forces $\vec{F_1} = 2\hat{i} + 3\hat{j}$ N $\vec{F_2} = (3\hat{i} + 2\hat{j})$ N acting on $M_1 = 2$ kg & $m_2 = 3$ kg Find ~~at~~ Mag. of acc. of COM

Sol.

$$\vec{a}_{CM} = \frac{\vec{F_1} + \vec{F_2}}{m_1 + m_2}$$

$$\Rightarrow \frac{2\hat{i} + 3\hat{i} + 3\hat{i} + 2\hat{j}}{5}$$

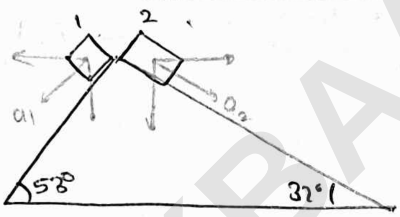
$$\Rightarrow \frac{5\hat{i} + 5\hat{j}}{5}$$

$$= \sqrt{2} \text{ m/s}^2$$

$\frac{5\sqrt{2}}{3}$

Good

Ques. Two Blocks of equal masses are released from rest, from top of an inclined as shown. find Mag. of acc. of COM ?



$$a_{cm} = \frac{\vec{F_1} + \vec{F_2}}{m_1 + m_2}.$$

$$= \cancel{m_1 g \sin\theta + mg}$$

$$\Rightarrow \cancel{\frac{g\frac{4}{5} + g\left(\frac{3}{5}\right)}{2m}}$$

$$\Rightarrow \cancel{g\left(\frac{7}{5}\right)}$$

Mistake

$\frac{16g^2}{25} + \frac{9g^2}{25}$

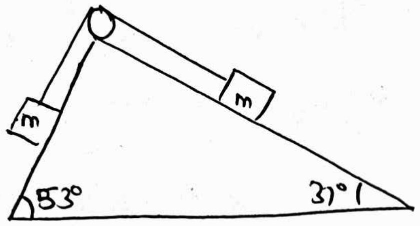
$a_1 = g \sin 53° = \frac{4g}{5}$ $a_2 = g \sin 37° = \frac{3g}{5}$

$$\vec{a}_{cm} = \frac{m\vec{a_1} + m\vec{a_2}}{2m} = \frac{\vec{a_1} + \vec{a_2}}{2}$$

$$\boxed{|\vec{a}_{cm}| = \frac{1}{2}\sqrt{a_1^2 + a_2^2}}$$ If masses are equal.

$$= \frac{g}{2}$$

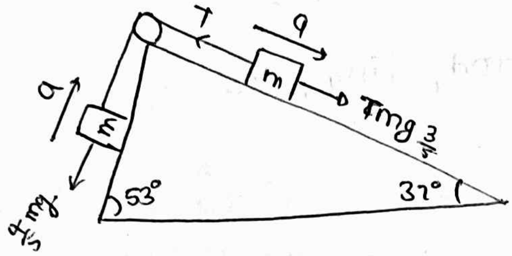
Que



$|\vec{a}_{cm}| = ?$

$\rightarrow \dfrac{\vec{a}_1 - \vec{a}_2}{2}$

sol.



Both block will move with same a.

$T - \frac{3mg}{5} - T = ma$

$\frac{4m}{5} - T - 4m$

$\frac{mg}{5} = 2a$

$8m - T = ma$

$T - 6m = ma$

$2m = ma$

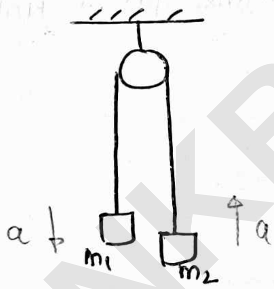
$a = 2$

$|\vec{a}_{cm}| = \frac{1}{2}\sqrt{a_1^2 + a_2^2} = \frac{1}{2}\sqrt{2}a = \frac{a}{\sqrt{2}}$

$\Rightarrow \frac{g}{5\sqrt{2}}$

Que.

$m_1 > m_2$



$~~|\vec{a}_{cm}| ?~~$

$\vec{a}_{cm} = ?$

sol.

$|\vec{a}_1| = |\vec{a}_2| = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$

$a_1 = \frac{m_1\vec{a}_1 - m_2\vec{a}}{m_1 + m_2}$

$\vec{a}_{cm} = \dfrac{m_1(-a\hat{j}) + m_2(a\hat{j})}{m_1 + m_2}$

$\Rightarrow \left(\frac{m_2 - m_1}{m_1 + m_2}\right) a\hat{j}$

$$\boxed{\vec{a}_{cm} = -\left[\frac{m_1 - m_2}{m_1 + m_2}\right]^2 g\,\hat{j}}$$

COM towards Heavier one

CONCEPT OF MASS-DENSITY

* linear Mass density (λ) :-

if λ + L same

$$\vec{r}_{cm} = \frac{\vec{r}_1 + \vec{r}_2 + \dots + \vec{r}_n}{n}$$

- $\lambda = \frac{dm}{dl}$ or $\frac{M}{L}$
- used for thin wire, thin rod, ring, etc.
- unit → kg/m
- $\boxed{\int dm = \int \lambda \, dl}$ (λ = variable) OR $\boxed{M = \lambda L}$ (λ = const.)

* surface Mass density (σ) :-

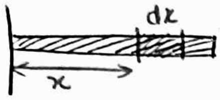
- $\sigma = \frac{dm}{dA}$ OR $\frac{M}{A}$
- used for Disc, lamina, plate, Hollow sphere, Hollow cylinder
- unit → kg/m²
- $\boxed{\int dm = \int \lambda \cdot dA}$ (σ = variable) OR $\boxed{M = \sigma A}$ $\boxed{\sigma = \text{constant}}$

* volume Mass-Density (ρ) :-

- $\rho = \frac{dm}{dv}$ OR $\frac{M}{V}$
- used for solid sphere, cube, solid cylinder
- unit → kg/m³
- $\boxed{\int dm = \int \rho \cdot dv}$ OR $\boxed{M = \rho V}$

Que. Find the Mass of the rod having ~~constant~~ linear mass density $= \lambda_0 x$
Length $= L$

sol.



$\lambda = \lambda_0 x$

~~dm~~ $\frac{dm}{dx} = \lambda_0 x$

$dm = \lambda_0 x \cdot dx$

$M = \int dm$

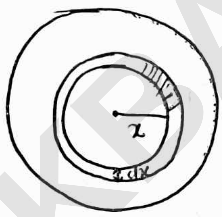
$= \int_0^L (\lambda_0 x) \cdot dx$

$\Rightarrow \lambda_0 \left[\frac{x^2}{2}\right]_0^L$

$\boxed{M = \frac{\lambda_0 L^2}{2}}$

Que. Find mass of disc having const. σ 4 r

sol.



Area = length x thickness

vol. = length x (surface Area or cross section)

$dm = \sigma (dA) = \sigma (\underbrace{2\pi x \cdot dx}_{dA})$

$dA = (2\pi x) dx$

$\downarrow$

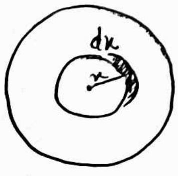
$A = \int dA = \int_0^R 2\pi x \cdot dx = \pi R^2$

$\boxed{M = \sigma (\pi R^2)}$

Volume of sphere

$\downarrow$

elemental vol. $\quad dV = (4\pi x^2) dx$



$\Rightarrow$

$V = \int dV = \int_0^R \frac{4\pi x^3}{3} = \frac{4\pi R^3}{3}$

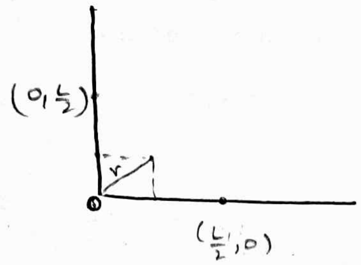
$\Rightarrow \quad dm = \rho \cdot dv$

$$\Rightarrow M = \int dm$$

$$\Rightarrow \rho \cdot \int_0^R 4\pi x^2 \cdot dx$$

$$\boxed{M \Rightarrow \rho \frac{4}{3}\pi R^3}$$

Ques. Two rods of same length & same const. λ locate COM



Sol.

$M_1 = \lambda L$

$M_2 = \lambda L$

$r = \frac{M_1 r_1 + M_2 r_2}{M_1 + M_2}$

$\Rightarrow \lambda L$

$M_1 r_1 =$

$$x_{cm} = \frac{M\left(\frac{L}{2}\right) + M(0)}{2M} = \frac{L}{4}$$

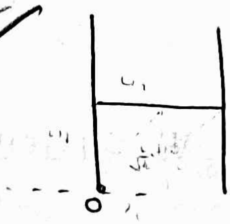
$$y_{cm} = \frac{M(0) + M\left(\frac{L}{2}\right)}{2M} = \frac{L}{4}$$

At a distance $\frac{L}{4}\sqrt{2}$ from point of intersect^n (point where rods are joined)

Method 2

$$\vec{r}_{cm} = \frac{\vec{r}_1 + \vec{r}_2}{2} = \frac{\frac{L}{2}\hat{i} + \frac{L}{2}\hat{j}}{2} = \frac{L}{4}(\hat{i}+\hat{j}) = \frac{L}{4}\sqrt{2}$$

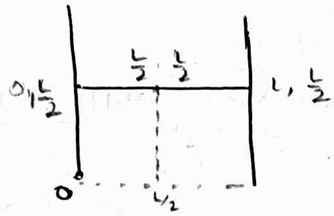
Ques.



equal mass, equal length, 3 rods.

$r_{cm} = ?$

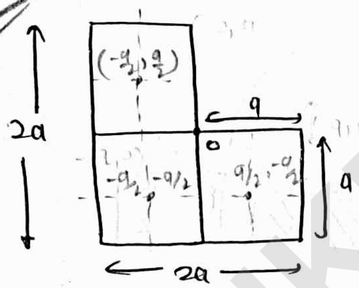
Sol.



$$x_{cm} = \frac{M(0) + M(\frac{L}{2}) + ML}{3M} = \frac{L}{2}$$

$$y_{cm} = \frac{M(\frac{L}{2}) + M(\frac{L}{2}) + M(\frac{L}{2})}{3M} = \frac{L}{2}$$

Ques.



$r_{cm} = ?$

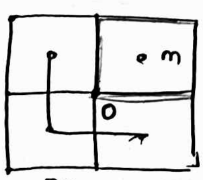
Sol.

$$x_{cm} = \frac{M(-\frac{a}{2}) + M(-\frac{a}{2}) + M(\frac{a}{2})}{3M} = -\frac{a}{6}$$

$$y_{cm} = \frac{M(\frac{a}{2}) + M(-\frac{a}{2}) + m(-\frac{a}{2})}{3M} = -\frac{a}{6}$$

$$r_{cm} = -\frac{a}{6}, -\frac{a}{6}$$

Alternate



3m

$$0 = \frac{m(\frac{a}{2}) + 3m(x)}{4m} \Rightarrow x = -\frac{a}{6}$$

$$0 = \frac{m(\frac{a}{2}) + 3m(y)}{4m} \Rightarrow y = -a/6$$

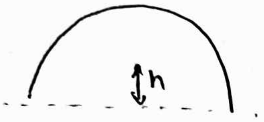
~~Ques~~

LOCATION OF CENTRE OF MASS FOR SYSMETRIC BODIES OR SYSTEM OF PARTICLE

① It must be at centre of symmetry or at Geometrical centre.

② **Half Bodies**

ⓐ **Half Ring**



$\boxed{h = \frac{2R}{\pi}}$

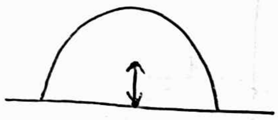
ⓑ **Half disc**

$\boxed{h = \frac{4R}{3\pi}}$

ⓒ **Half sphere**

$\boxed{h = \frac{3R}{8}}$

ⓓ **Half shell**



$\boxed{\cancel{h = \frac{3R}{8}}}$ $\boxed{h = \frac{R}{2}}$

S R HD

$\frac{1}{2}$ $\frac{2}{\pi}$ $\frac{3}{8}$ $\frac{4}{3\pi}$

Ques.

Sq. plate and a Half Disc is joined as shown

$\sigma = \star$

COM will lie
1) in plate
2) in disc
3) at line joining them
4) NOTA

Sol.

$C_1 = -R, 0$

$C_2 = \frac{4R}{3\pi}, 0$

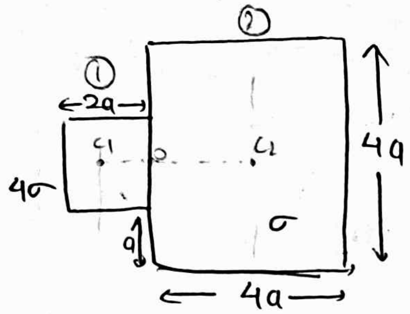
$$r_{cm} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

→ +ve → Disc
→ −ve → Plate
→ 0 → origin.

$$= 4R^2(-R) + \frac{4R}{3\pi}\left(\frac{\pi R^2}{2}\right)$$

$$= -4R^3 + \frac{2R^3}{3} = -ve$$

Ques.



TRICK

(Mx) का Magnitude जिसका Greater COM उसकी तरफ

$c_1 = -a, 0$

$c_2 = 2a, 0$

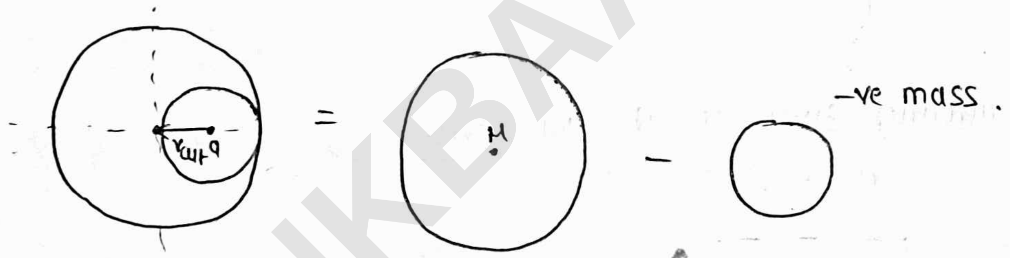
$$r_{cm} = \frac{A_1\sigma_1 x_1 + A_2\sigma_2 x_2}{A_1\sigma_1 + A_2\sigma_2}$$

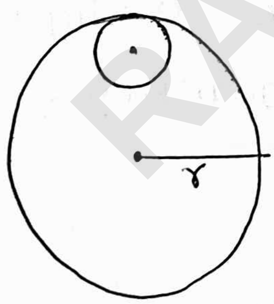
$$= \frac{4a^2 \times 4 \times -a + 16a^2 \times 1 \times 2a}{16a^2 + 16a^2}$$

$$= \frac{-16a^3 + 32a^3}{32a^2} = \underline{\underline{+ve}}$$ Bigger Plate

CONCEPT OF MASS REMOVED :-

A.





r = remaining

c = cut

~~$M_{rem}\, r_{rem}$~~

$m_r r_r + m_c r_c = 0$

$$\boxed{r_r = -\left[\frac{M_{cut}}{M_{rem}}\right] r_{cut}}$$

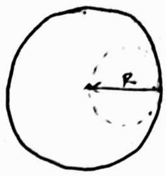
Disc/Plate → Area

Sphere → Vol.

$$\boxed{r_{rem} = -\left[\frac{A_{cut}}{A_{rem}}\right] r_{cut}}$$

$$\boxed{r_{rem} = -\left[\frac{V_{cut}}{V_{rem}}\right] r_{cut}}$$

DISC



$A_{cut} = \pi\left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4}$

$A = \pi R^2$

$A_{rem} = \frac{3\pi R^2}{4} = A - A_{cut}$

$r_{rem} = -\left[\frac{1}{3}\right]\left[\frac{R}{2}\right] = -\frac{R}{6}$

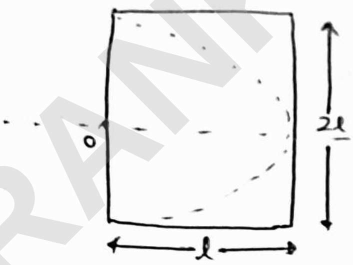
VOLUME

$V = V$ $\quad V_{cut} = \frac{V}{8}$ $\quad V_{rem} = \frac{7V}{8}$

$r_{rem} = -\left(\frac{1}{7}\right)\left(\frac{R}{2}\right)$

$\Rightarrow -\frac{R}{14}$

If origin is already given in the question:-



CM of remaining mass = ?

semi-circular disc is being cut from a rectangular plate

Sol.

CM for complete body $= \left(\frac{l}{2}, 0\right)$

$A = (2l)(l) = 2l^2$

$A_{cut} = \frac{\pi l^2}{2}$

$A_{rem} = 2l^2 - \frac{\pi l^2}{2} = \frac{(4-\pi)l^2}{2}$

$$\Rightarrow \quad x_{cm} = \frac{M_1x_1 + M_2x_2}{M_1+M_2}$$

$$\Rightarrow \quad x_{cm} = \frac{A_1x_1 + A_2x_2}{A_1+A_2}$$

$$\Rightarrow \quad \frac{l}{2} = \frac{\left(\frac{\pi l^2}{2}\right)\left(\frac{4l}{3\pi}\right) + \left(\frac{4-\pi}{2}\right)l^2 \cdot x_2}{2l^2}$$

$$\Rightarrow \quad l^3 = \frac{2}{3}l^3 + \left(\frac{4-\pi}{2}\right)l^2 \cdot x_2$$

$$\Rightarrow \quad \frac{l^3}{3} = \left(\frac{4-\pi}{2}\right)l^2 \cdot x_2$$

$$\Rightarrow \quad \frac{2l}{3(4-\pi)} = x_2$$

$$\Rightarrow \quad \boxed{x_2 = \frac{2l}{3(4-\pi)}}$$

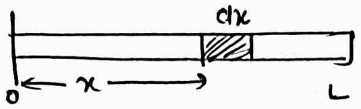
COM using integration

* used for non-uniform mass distribution generally.
* continuous mass distribut^n

$$\boxed{r_{cm} = \frac{\int r \cdot dm}{\int dm}} = \frac{\int r \cdot dm}{M}$$

$$x_{cm} = \frac{\int x \cdot dm}{\int dm} \qquad y_{cm} = \frac{\int y \cdot dm}{\int dm} \qquad z_{cm} = \frac{\int z \cdot dm}{\int dm}$$

$\lambda = \lambda_0$ (constant) for a rod of length 'L'



$$\boxed{dm = \lambda dx}$$

$$x_{cm} = \frac{\int_0^L x \cdot (\lambda dx)}{\int_0^L \lambda \cdot dx} = \frac{\lambda_0 (L^2/2)}{\lambda_0 (L)} = \frac{L}{2}$$

$\lambda = \lambda_0 x$ where x is dist. from one end.

$$dm = \lambda \cdot dx$$

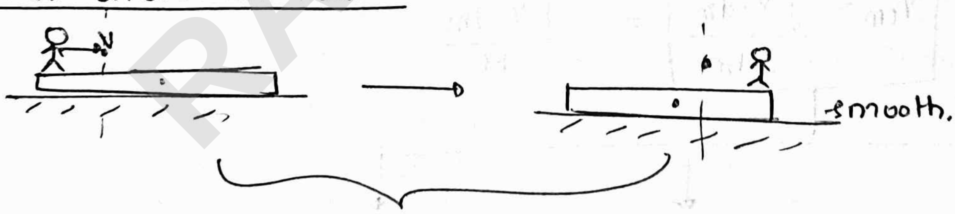
$$x_{cm} = \frac{\int_0^L (\lambda_0 x \cdot dx) x}{\int_0^L \lambda_0 x \cdot dx} = \frac{\lambda_0 [L^3/3]}{\lambda_0 [L^2/2]} = \frac{2L}{3}$$

IMP. POINTS

* Motion of CM will be unaffected if $\vec{F}_{ext}$ on the system $= 0$ (then CM will follow law of inertia)

* When particle moves under mutual attract^n or repulsion starting from rest.

Ⓐ Person on wooden plank



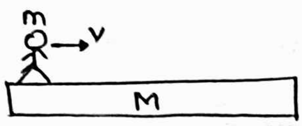
COM will be at rest.

$\vec{F}_{ext} = 0$

COM at rest Always.

Ⓑ

Que A Person is moving on A wooden Plank towards other end, when plank is kept on smooth surface.



① Person moving with v w.r.t Plank

$$\vec{V}_{mp} = \vec{V}_m - \vec{V}_P$$

Acc. to Momentum conservation (w.r.t ground)

$$0 = \vec{V}_m m + M\vec{V}_P$$

$$\vec{V}_P = \frac{-m\vec{V}_m}{M}$$

$$0 = m(\vec{V} + \vec{V}_P) + M\vec{V}_P$$

$$\boxed{\vec{V}_P = \frac{-m\vec{V}}{m+M}}$$

② Person moving with v w.r.t ground

$$\vec{V}_{mp} = \vec{V}_M - \vec{V}_P \qquad \vec{V}_m = V$$

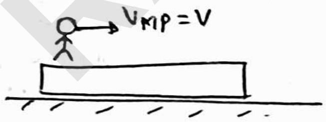
$$\vec{V}_P = -\frac{m}{M}\vec{V}_m$$

$$\boxed{\vec{V}_P = -\frac{m\vec{V}}{M}}$$

* Time taken by Person to Reach to other end

$$\boxed{t = \frac{L}{\vec{V}_{Mp}}}$$

*

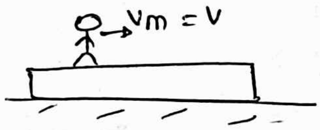


$$V_P = -\frac{mV}{m+M}$$

$$t = \frac{L}{V}$$

* shift in Plank to keep CM at same position

$$\boxed{x = \frac{-mL}{M+m}}$$



* $\cancel{V_P = -\frac{MV}{M}}$

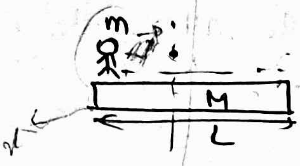
* $V_P = \frac{-mV}{M}$

* $t = \frac{ML}{(m+M)V}$

* Shift In Plank

$$\boxed{x = -\frac{mL}{M+m}}$$

Ques.



$F_{ext} = 0$

$x_a =$

sol.

$$M_1x_1 = M_2x_2.$$

$$m(L-x) = Mx$$

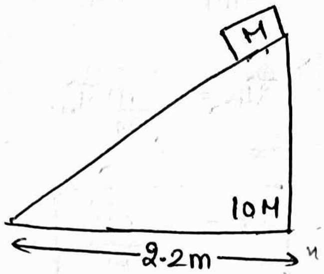
$$\boxed{x = \frac{mL}{M+m}}$$

$x_1 = 0$

$\frac{m_1x_1 + m_2x_2}{m_1 + m_2} =$

$m_1x_1 = m_2x_2$

Ques.



Distance travelled by wedge when Block of Mass M reaches to bottom.

$m_1x_1 = m_2x_2$

$Mx = 10M(2.2 + x)$

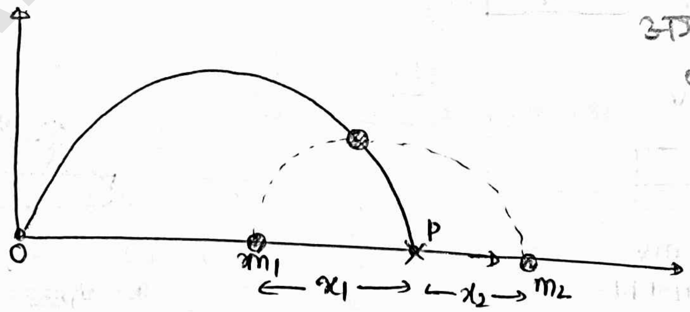
$x = 22 + 10x$

sol.

$$M(2.2 - x) = 10Mx.$$

$$x = \frac{2.2}{11} = \cancel{0.2m}\ 0.2m$$

NOTE: When a particle is projected + it get exploded in two or More fragements then the Motion of COM will always on same trajectory until all fragements reaches to ground



अगर bomb rest पर explode हुआ था तो COM will be always at rest.

$P \rightarrow$ origin

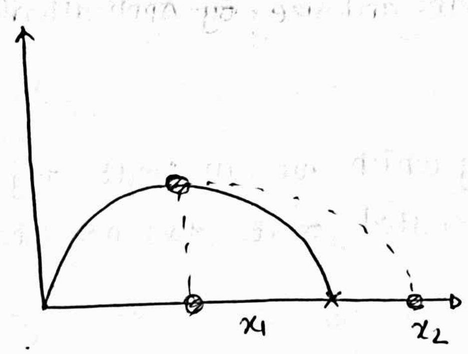
$$\boxed{M_1x_1 = M_2x_2}$$

$O \rightarrow$ origin

$$\boxed{R = \frac{M_1(R-x_1) + M_2(R+x_2)}{M_1 + M_2}}$$

Ques. A Bomb is projected with spd 100m/s at angle 37° to Horizontal at highest point it explodes in two fragements of mass ratio 1:3 such that smaller part comes to rest instantly. Find dist of Point w.r.t Point of Project^n where 2nd part hits ground

Sol.



$R = \frac{u^2 \sin 2\theta}{g}$

$\frac{2 \times 100 \times 100 \times \frac{4}{5} \times \frac{3}{5}}{10}$

24×40

960 m.

$M_1x_1 = M_2x_2$

$1 \times 480 = 3 \times x$

$x = 160$

$960 + 160 = 1120$ m

$M_1x_1 = M_2x_2$

$m\left(\frac{R}{2}\right) = 3m(x_2)$

$x_2 = \frac{R}{6}$

from origin $= R + \frac{R}{6} = \frac{7R}{6}$

$R = \frac{u^2 \sin 2\theta}{g} = \frac{100 \times 100 \times 2 \times \frac{4}{5} \times \frac{3}{5}}{10} = 960$

$= \frac{7}{6} \times 960 = 1120$m.

ROTATIONAL MOTION

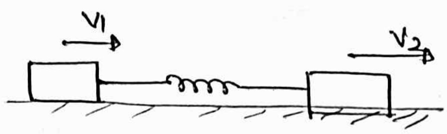
L-1

* Rigid Body :-

- A body which can't change its shape and size by Application of external Force and torque.

- It's a Mathematically concept by which we can treat any body as rigid body if dist. b/w any 2 point located on it does not change

ex.



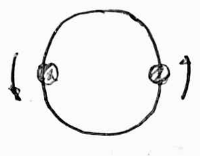
if $v_1 = v_2$ R.B

if $v_1 \neq v_2$ Not. R.B

Rigid Body

उस Body के Any point do not change their positn उस body को Rigid body है।

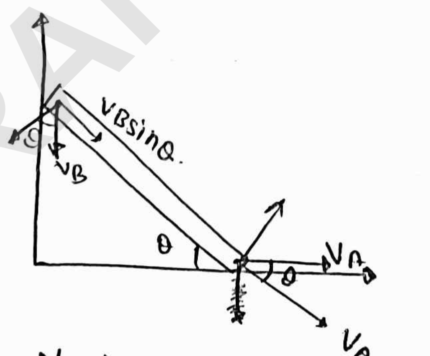
Ex.



$v_1 = v_2$ R.B

$v_1 \neq v_2$ Not a R.B

E.x



$v_B \sin\theta = v_A \cos\theta$

$\boxed{\frac{v_A}{v_B} = \tan\theta}$ Neet PYQ

MOMENT OF INERTIA :-

* It is a Tensor quantity which is ~~not~~ neither scalar nor vector.
* value of MOI does not depend on dirⁿ, it depends on
  - $\boxed{\text{Axis of rotation}}$
  - Mass of body
  - Mass distribution around axis of rotations.
  - ⊥ar dist. of mass w.r.t axis of rotations

  } Moment of mass system.

1) For a particle $\boxed{I = Mr^2}$

2) For a system $\boxed{I = M_1r_1^2 + M_2r_2^2 \dots + m_nr_n^2}$

$\boxed{I = \Sigma(Mr^2)}$

3) For Rigid Body $\boxed{I = \int (dm) r^2}$

* Radius of Gyration (K) :-

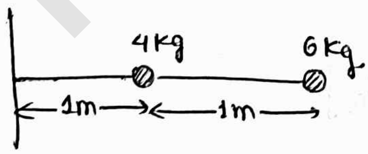
$I = MK^2$

POM $\neq$ $\boxed{K = \sqrt{\frac{I}{M}}}$

system of Particle → $K = \sqrt{\frac{\Sigma(mr^2)}{\Sigma(m)}}$

Rigid Body → $K = \sqrt{\frac{\int r^2 dm}{\int dm}}$

case

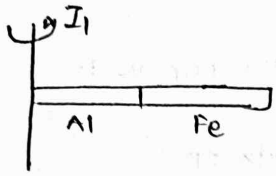


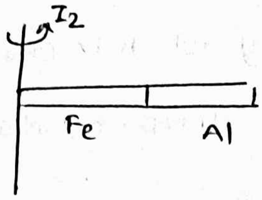
$I = 4(1)^2 + 6(2)^2 = 28 \text{ kg m}^2$

$I = 6(1)^2 + 4(2)^2 = 22 \text{ kg m}^2$

* $\boxed{\text{'I' will be large when heavier mass is far from Axis of Rotation.}}$

Ques.





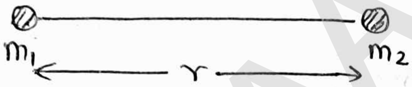
$I_1 > I_2$.

* If a particle lies on Axis of rotation then its moment of inertia equal to zero.

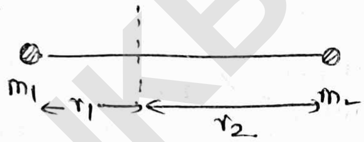
$\boxed{I_{axis} = 0}$

for remaining all $r = \perp$ar dist from AOR.

Ques. Find moment of inertia of a system of two particles kept at a dist. R about an axis passing through their COM & $\perp$ar to line joining ~~jet~~ them.



Sol.



$$r_1 = \frac{m_2 r}{m_1 + m_2} \qquad r_2 = \frac{m_1 r}{m_1 + m_2}.$$

$$I = m_1 r_1^2 + m_2 r_2^2$$

$$= m_1 \left[\frac{m_2 r}{m_1 + m_2}\right]^2 + m_2 \left[\frac{m_1 r}{m_1 + m_2}\right]^2$$

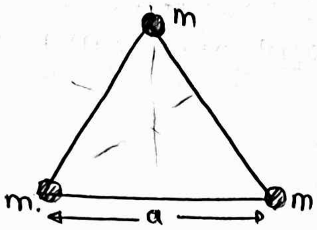
$$\boxed{I \Rightarrow \left(\frac{m_1 m_2}{m_1 + m_2}\right) r^2}$$

$\boxed{I = mr^2}$

concept of Reduced Mass

$\boxed{m = \frac{m_1 m_2}{m_1 + m_2}} \Rightarrow$ Reduced Mass

Ques.



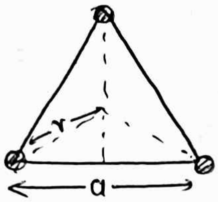
i) Find M.I about an axis passing through centeriod & $\perp$ar to plane of Δ. & also (K)

ii) about an axis passing through one of its vertices & $\perp$ar to plane also (K).

iii) about an axis passing through one of its vertices & in the plane of Δ. & (K)

Sol.

i)



$r = \frac{a}{\sqrt{3}}$

$I = 3mr^2$

$\Rightarrow 3m\left[\frac{a}{\sqrt{3}}\right]^2$

$\boxed{I \Rightarrow ma^2}$

$I = 2ma^2$

$K = \sqrt{\frac{I}{m}} = \sqrt{\frac{ma^2}{3m}} = \frac{a}{\sqrt{3}}$

ii)

$I = ma^2 + ma^2 = 2ma^2.$

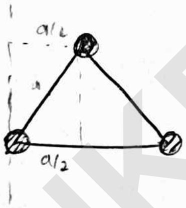
$K = \sqrt{\frac{2ma^2}{3m}} = \sqrt{\frac{2}{3}}a$

mistake $m' = 3m$ होगा है

Mistake

$K = I$ नहीं करना है directly

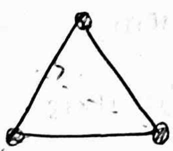
iii)



$I = ma^2 + m\frac{a^2}{4} = \frac{5ma^2}{4}$

$K = \sqrt{\frac{5ma^2}{4(3m)}} = \sqrt{\frac{5}{12}}\,a$

iv) About an axis along one of its sides.

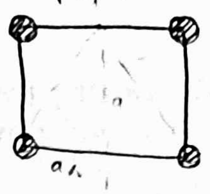


$I = ma^2 = m\left(\frac{\sqrt{3}a}{2}\right)^2 = \frac{3ma^2}{4}$

$K = \sqrt{\frac{\cancel{m}a^2}{\cancel{3m}}} = \frac{a}{\sqrt{3}}.$

$K = \sqrt{\frac{3ma^2}{4 \cdot 3m}} = \frac{a}{2}$

Que. Axis of rotation passing through mid point of parallel sides + in the plane of square



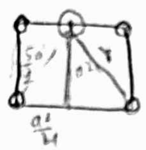
Find M.I

Sol.

$$I = 4m\left(\frac{a}{2}\right)^2 = ma^2$$

$$K = \sqrt{\frac{ma^2}{4m}} = \frac{a}{2}.$$

ii) Axis passing through mid point of one of sides + ⊥ar to the plane of square



$$I = 2mr^2 + 2m\left(\frac{a}{2}\right)^2$$

$$= 2m\left(\frac{\sqrt{5}}{g}a\right)^2 + \frac{2ma^2}{2}$$

$$\Rightarrow 3ma^2$$

$$K = \sqrt{\frac{3ma^2}{4m}} = \sqrt{\frac{3}{4}}\,a$$

Que. 3 particles of masses 1 kg, 2 kg + 3 kg are situated at (1,1,0) (2,0,0) + (0,3,0) respectively. Find M.I of system

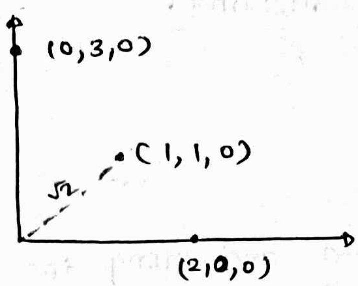
i) About x axis ii) Y axis iii) z-axis

i) $I = 1\times(1)^2 + 2(2)^2 + 3(0)^2$
$\Rightarrow 1 + 8$
$= 9\ \text{kg}m^2$

ii) $I = 1\times(1)^2 + 2(2)^2 + 3(3)^2$
$= 1 + 8 + 27$
$\Rightarrow 36\ \text{kg}m^2$

iii) z-axis $I = 1(0)^2 + 2(0)^2 + (0)^2 3$
$= 0$

Sol.



$I_x = 1(1)^2 + 2(0)^2 + 3(3)^2 = 1 + 27 = 28$ kgm²

$I_y = 1(1)^2 + 2(2)^2 + 3(0)^2 = 1 + 8 = 9$ kgm²

$I_z = 1(\sqrt{2})^2 + 2(2)^2 + 3(3)^2 = 2 + 8 + 27 = 37$ kgm².

⊥ar distances of a point (x_1, y_1) from a straight line.

1) $y = mx + c$

$mx - y + c = 0$

$$\boxed{r_\perp = \frac{mx_1 - y_1 + c}{\sqrt{m^2+1}}}$$

not mass it is slope.

2) $ax + by + c = 0$

$$\boxed{r_\perp = \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}}$$

Ques. cal. MI of a particle located at (2,2) w.r.t an axis $y = 2x+3$
$m = 3$kg

Sol.

$2x - y + 3 = 0$

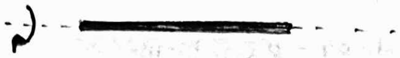
$$r_\perp = \frac{4-2+3}{\sqrt{4+1}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

$$I = mr^2 = 3 \times (\sqrt{5})^2 = 15 \text{ kg m}^2$$

CONTINOUS MASS DISTRIBUTION (RIGID BODY)

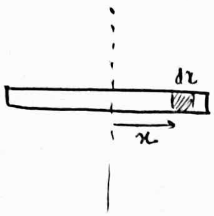
① ROD (mass m , length L)
↓
Thin rod

i) About an axis passing through COM and along the length.



$r = 0$
$I = 0$

ii) About an axis passing through COM & ⊥ar to length



$$dm = \frac{M}{L} \cdot dx$$

$$I = \int x^2 \cdot dm$$

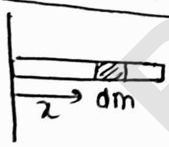
$$= \frac{M}{L}\int_{-L/2}^{L/2} x^2 \cdot dx$$

$$\Rightarrow \frac{M}{3L}\left[x^3\right]_{-L/2}^{L/2}$$

$$= \frac{M}{3L}\left[\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right]$$

$$\boxed{I = \frac{ML^2}{12}}$$

iii) About an axis passing through ~~com~~ one of its end & ⊥ar to length



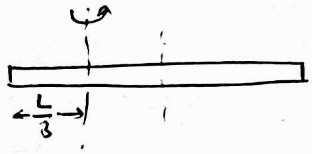
$$I = \int x^2 \cdot dm$$

$$\Rightarrow \int_0^L \frac{M}{L} x^2 \cdot dx$$

$$\Rightarrow \frac{M}{3L}\left[L^3\right]_0^L$$

$$\boxed{I \Rightarrow \frac{ML^2}{3}}$$

Ques.

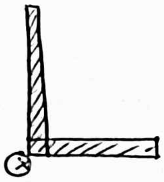


$$I = \frac{M}{3L}\left[x^3\right]_{-L/3}^{2L/3} = \frac{M}{3L}\left[\frac{8L^3}{27} + \frac{L^3}{27}\right]$$

$$= \frac{M}{3L} \times \frac{9L^3}{27}$$

$$= \frac{ML^2}{9}.$$

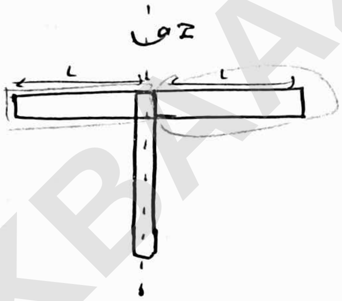
Ques.



$M \cdot I = ?$

$$I = \frac{ML^2}{3} + \frac{ML^2}{3} = \frac{2ML^2}{3}.$$

Ques.

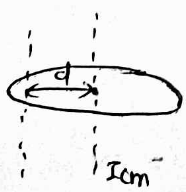


$$I = \frac{ML^2}{3} + \frac{ML^2}{3} + 0 = \frac{2ML^2}{3}.$$

PARALLEL AXIS THEOREM

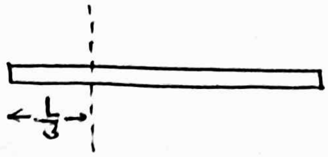
* Applicable for each kind of bodies. (2D, 3D).

* Two axes
- Mutually parallel
- one axis → CM से Pass हो
- 2nd axis → जिसके about MI calculate करना है।

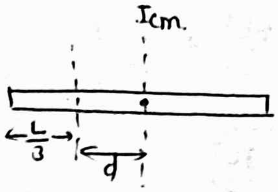


$$\boxed{I = I_{cm} + Md^2}$$

Ques.



Sol.

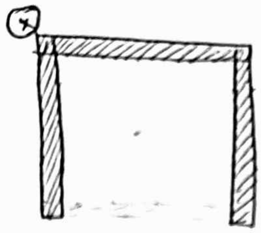


$$d = \frac{L}{2} - \frac{L}{3} = \frac{L}{6}.$$

$$I = I_{cm} + Md^2 .$$

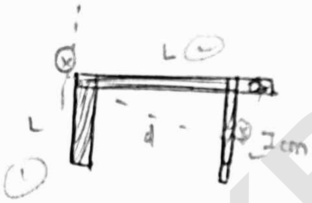
$$\Rightarrow \frac{ML^2}{12} + M\left(\frac{L^2}{36}\right) = \frac{ML^2}{9}.$$

Ques.



M.I = ?

Sol.

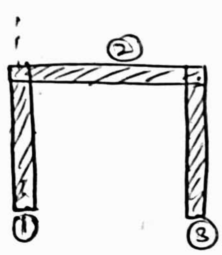


$$I_1 = I_2 = \frac{ML^2}{3}$$

$$I_3 = \frac{ML^2}{12} + Md^2.$$

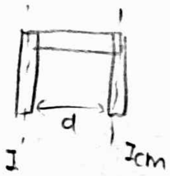
$$I = I_1 + I_2 + I_3$$

Ques.



$I_1 = 0$

$I_2 = M$



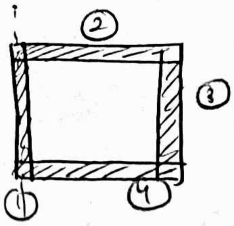
$$I_1 = 0$$

$$I_2 = \frac{ML^2}{3}$$

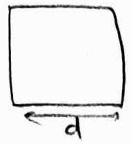
$$I_3 = 0 + Md^2 = mL^2$$

$$\boxed{I = \frac{4}{3}ML^2}$$

Ques.



sol.



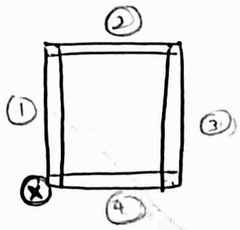
$I_1 = 0$

$I_2 = I_4 = \frac{ML^2}{3}$

$I_3 = 0 + ML^2$

$I = \left(\frac{2}{3}+1\right)ML^2 = \frac{5}{3}ML^2$

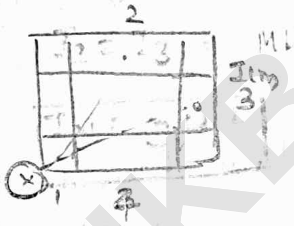
Ques.



About an axis passing through one of vertices + ⊥ar to Plane.

sol.

$I_1 = I_4 = \frac{ML^2}{3}$



$I_{HX} = I_{cm} + Md^2$

$\Rightarrow \frac{ML^2}{12} + M\frac{5L^2}{4}$

$\Rightarrow \frac{16ML^2}{12}$

$I_2 = I_3 \Rightarrow \frac{4}{3}ML^2$

PERPENDICULAR AXIS THEOREM

* **Three Axes** → तीनों ही Mutually ⊥ar होनी हैं। And all three have a common point of intersection.

  • two axes must lie in plane of body.
  I_1 I_2

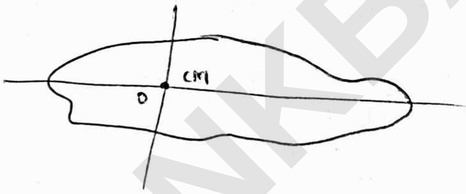
* Applicable For planar bodies.
  eg:- Ring, Disc, Plate.

  • 3rd Axes → must be ⊥ar to plane of body.
  + I_3.

then $\boxed{I_3 = I_1 + I_2}$

Body in x-y Plane	Body in yz Plane	Body is xz Plane
$I_z = I_x + I_y$	$I_x = I_y + I_z$	$I_y = I_z + I_x$.

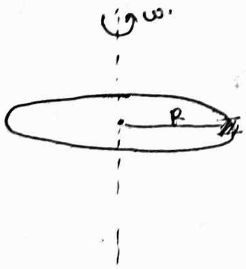
ex:-



3rd axis → Passing through the point O + ⊥ar to the plane.

① RING:-

Ⓐ About its Geometrical Axis (About an axis passing through its COM + ⊥ar to the plane)

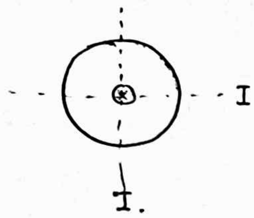


$dI = \int dm R^2$.

$I = \int dI = R^2 \int dm = MR^2$

$\boxed{I = MR^2}$

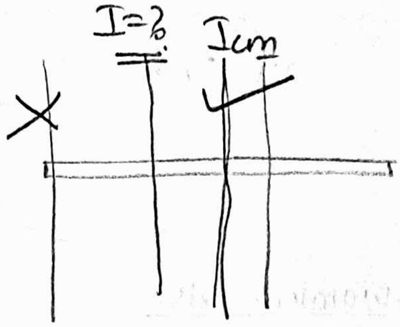
(B) About an axis passing through CM and in plane of the ring
OR
About its diametric axis.



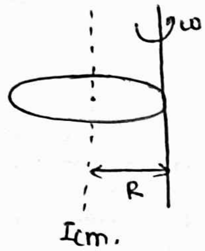
$I_{geo} = I_{dia} + I_{dia}$

$I_d = \frac{I_g}{2}$

$\boxed{I_d = \frac{MR^2}{2}}$



(C) About a tangent ⊥ lar to the plane



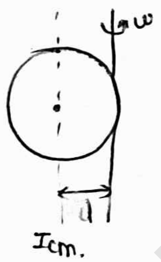
From $\parallel^{e}$ axes theorem.

$I = I_{cm} + Md^2$

$= MR^2 + M(R)^2$

$\boxed{I = 2MR^2}$

(D) About an axis along the tangent and in plane of the body.

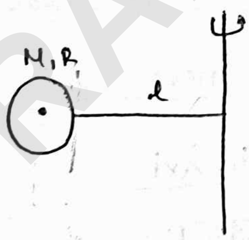


$I = I_{cm} + Md^2$

$= \frac{MR^2}{2} + MR^2$

$\boxed{I = \frac{3}{2}MR^2}$

Ques



$I = I_{cm} + Md^2$

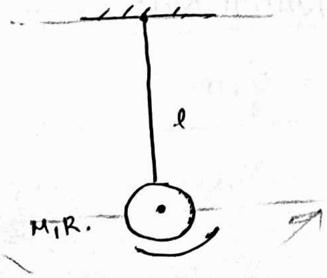
$\Rightarrow \frac{MR^2}{2} + M(l+R)^2$

M.I जिस Axis पर Find करना है उसको Icm के respect में ही लेना है।

$M\left(\frac{3R^2}{2} + l^2\right)$

$M(R^2 + l^2 + 2R)$

Ques.



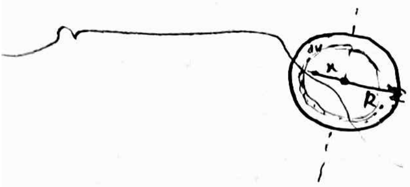
I=? About an axis passing through pos ⊥ lar to the plane of oscillations.

$I_{cm} = I_{geo} = MR^2$

$I = MR^2 + M(l+R)^2$

② Disc

Ⓐ About its Geometrical Axis

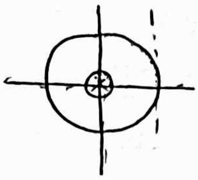


$dI = \int dm\, R^2$

$dm = \frac{M}{\pi R^2} \cdot dx$

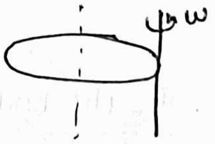
$I = \frac{MR^2}{2}$

Ⓑ Diametric Axis



$I_{dia} = \frac{MR^2}{4}$

Ⓒ Tangent ⊥ lar to plane



$I = \frac{MR^2}{2} + MR^2$

$= \frac{3}{2} MR^2$

Ⓓ Tangent ∥^{l} to Plane

$I = \frac{MR^2}{4} + MR^2$

$= \frac{5}{4} MR^2$

③ Solid sphere

Ⓐ Geometric Axis

$\boxed{I = \frac{2}{5} MR^2}$

Ⓑ Tangential Axis

$\boxed{I = \frac{7}{5} MR^2}$

④ Hollow sphere

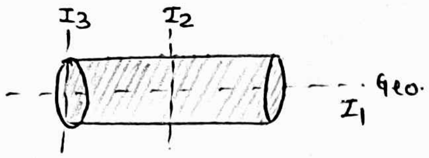
Ⓐ Geometric Axis

$\boxed{I = \frac{2}{3} MR^2}$

Ⓑ Tangential Axis

$\boxed{I = \frac{5}{3} MR^2}$

⑤ **Solid cylinder**



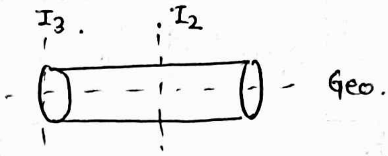
Ⓐ **Geometrical Axis**

$$I_1 = \frac{MR^2}{2}$$

Ⓑ $I_2 = \frac{MR^2}{4} + \frac{Ml^2}{12}$.

Ⓒ $I_3 = \frac{MR^2}{4} + \frac{Ml^2}{3}$

⑥ **Hollow cylinder**

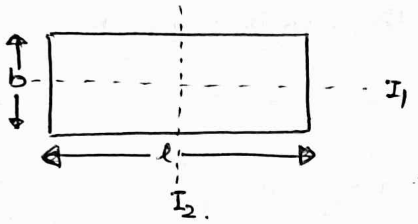


Ⓐ $I_1 = MR^2$

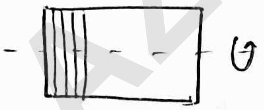
Ⓑ $I_2 = \frac{MR^2}{2} + \frac{Ml^2}{12}$

Ⓒ $I_3 = \frac{MR^2}{2} + \frac{Ml^2}{3}$.

⑦ **Rectangular plate :-**

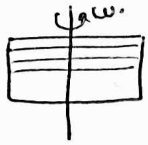


I_1 = M.I about an axis passing through cm & along the length.



$$I_1 = \frac{Mb^2}{12}$$

I_2 = About an axis passing through cm & along the breadth



$$I_2 = \frac{Ml^2}{12}$$

I_3 = About an axis passing through cm and ⊥ar to plane of the plate

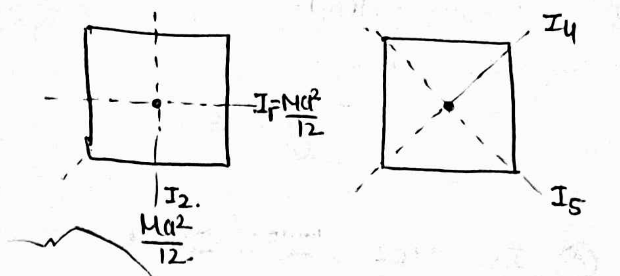
$$I_3 = I_1 + I_2 = I_3 = \frac{Mb^2}{12} + \frac{Ml^2}{12} = \boxed{\frac{M}{12}(l^2 + b^2)}$$

for square

$l = b = a$.

$$\boxed{I = \frac{Ma^2}{16}}$$

*



Diagonals of square are ⊥ar.

$I_1 = I_2 = \frac{I_3}{2}$

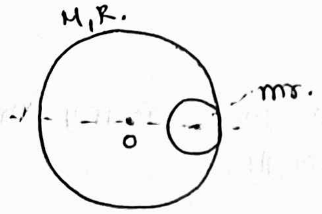
$I_4 = I_5$

$2I_4 = I_3$

$I_4 = \frac{I_3}{2}$

$$\boxed{I_1 = I_2 = I_4 = I_5 = \frac{Ma^2}{12}}$$

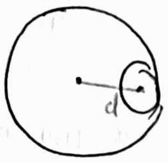
Ques.



M.I of remaining disc about an axis passing through o + ⊥ar to plane.

$R_{cut} = \frac{m}{M-m}(R-r)$

Sol.



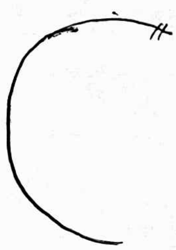
$$I = \frac{MR^2}{2} - \left[\frac{mr^2}{2} + md^2\right]$$

concept:- Mass is removed / added to a body.

① $I_{New} = I_{original} + I$ added mass about the same axis

② $I_{New} = I_{original} - I$ ~~added~~ removed mass about the same axis.

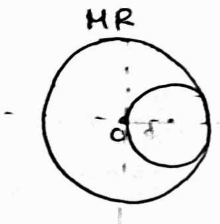
Ques.



$I_N = I_0 - I_{cut}$

$I_{cut} = I_{cm} + Ma^2$

Ques.



MI about an axis passing thrgh O.

$\frac{MR^2}{2} + \left[\frac{mr^2}{2} + \frac{MR^2}{4}\right]$

Sol.

$I_{original} = \frac{MR^2}{2}$

$I_{removed} = \frac{mr^2}{2} + mr^2$

$= \frac{3}{2} mr^2$

Here $r = \frac{R}{2}$ $\quad m = \frac{M}{\pi R^2} \pi r^2 \Rightarrow \frac{M}{R^2} r^2 = \frac{M}{4}$

$I_{New} = \frac{MR^2}{2} - \frac{3}{2}\left(\frac{M}{4}\right)\left(\frac{R^2}{4}\right]$

$= \frac{MR^2}{2} - \frac{3MR^2}{32}\Big]$

$\boxed{I \Rightarrow \frac{13MR^2}{32}}$

Ques. Advanced

M.I of an Annular disc of Mass = M, inner radius R_1, outer radius R_2 about geometrical axis

(A) $\frac{M}{2}(R_2^2 - R_1^2)$

(B) $\frac{M}{4}(R_2^2 - R_1^2)$

(C) $\frac{M}{2}(R_1^2 + R_2^2)$

(D) $\frac{M}{4}(R_1^2 + R_2^2)$

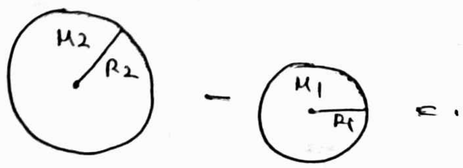
Sol.

$\frac{M_2R_2^2}{2} - \frac{M_1R_1^2}{2}$

$M_2 - M_1 = M$

$\sigma = \frac{M}{\pi(R_2^2 - R_1^2)} = \frac{M_2}{\pi R_2^2}$

$= \frac{M_1}{\pi R_1^2}$.



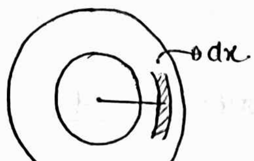
Sol.

$$I = \frac{M_2 R_2^2}{2} - \frac{M_1 R_1^2}{2}$$

$$= \frac{1}{2}\left[\frac{M R_2^2}{R_2^2 - R_1^2} \cdot R_2^2 - \frac{M R_1^2}{R_2^2 - R_1^2} \cdot R_1^2\right]$$

$$= \frac{M}{2(R_2^2 - R_1^2)}(R_2^4 - R_1^4) = \frac{M}{2}(R_2^2 + R_1^2)$$

Method 2



$$dm = \sigma\, 2\pi x \cdot dx.$$

$$\sigma =$$

$$dI = \int dm\, x^2$$

$$\sigma = \frac{M}{\cancel{\pi}(R_2^2 - R_1^2)}$$

$$dI = \sigma 2\pi \int_{R_1}^{R_2} x^3 \cdot dx$$

$$dI = \sigma 2\pi \left[\frac{x^4}{4}\right]_{R_1}^{R_2}$$

$$dI = \sigma 2\pi \frac{(R_2^4 - R_1^4)}{4}$$

$$= \frac{M(R_2^4 - R_1^4)}{2(R_2^2 - R_1^2)} = \frac{M}{2}(R_2^2 + R_1^2)$$

TORQUE

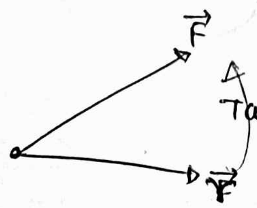
* It is defined as moment of force.

* $\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$

* $\boxed{\tau = rF\sin\theta}$ / $\boxed{\tau = Fr_\perp}$

$r_\perp$ = ⊥ar distance from axis of rotation to the line of action of the force.

* Diren :-



Fingers rolling towards $\vec{F}$

* Torque = Along the axis

↓

Axial vector.

↙ ⊗ = Inward ↘ ⊙ outward

* curling of Fingers = sense of rotation

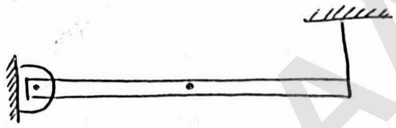
↙ CW ↘ ACW.

* For equilibrium condition force as well as torque must have to be zero.

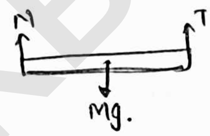
* For Transitional equilibrium. $F_{Net} = 0$

Rotational equilibrium $\tau_{Net} = 0$

Ques



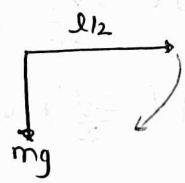
sol.



$N + T = Mg.$

Torque $\tau_N = N r_1 = 0$ (b'coz of $r_1 = 0$)

$\tau_{mg} =$



$\tau ⊗ = mg \frac{l}{2}$

$\tau_T =$

$\tau ⊙ = \cancel{mg}\ T \cdot l$

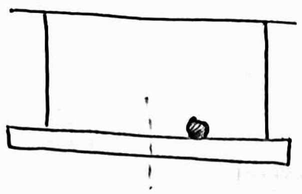
coz. $\tau ⊗ + \tau ⊙$ their magnit. are equal

$mg \frac{l}{2} = T l$

$\boxed{T = \frac{Mg}{2}}$

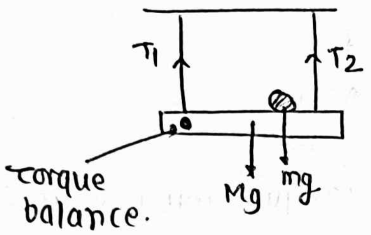
15/11/19

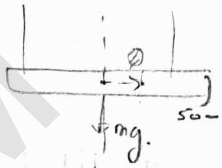
Ques.



one metre rod is suspended with help of 2 strings. Mass of rod = 50 gm, A stone of mass 10 gm is kept at a dist. 10cm from mid point. Find Tensions in the string.

sol.





$\tau =$ ~~mg + 10cm~~

$\tau > \frac{TL}{2}$.

$mg \times 10 = T \frac{50cm}{2}$

$T \to 10\times10^{-3} \times 2$

$2T = 60\times10^{-3}\times10$

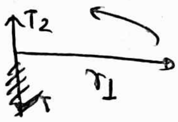
$2T \to 6\times10^{-1}$

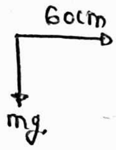
$T_1 + T_2 = Mg + mg.$

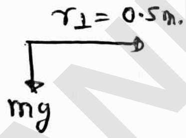
$T_1 + T_2 = 0.6.$

Torque balance.

$\tau_1 = 0.$

$\tau_2 =$  $= T_2 \odot$

$\tau_{mg} =$  $= 0.6mg$

$\tau_{Mg} =$  $= \frac{Mg}{2} \otimes$

$T_2 = \frac{Mg}{2} + 0.6mg$

$T_2 = g\left(\frac{25\times10^{-3}}{\cancel{2}} + 0.6\times10\times10^{-3}\right]$

$T_2 \Rightarrow g\left(\frac{25}{\cancel{2}} + 6\right] \times10^{-3}$

$T_2 \Rightarrow g\left(\cancel{\frac{25+12}{2}}\right)\cancel{10^{-3}}\quad 31\times10^{-2}.$

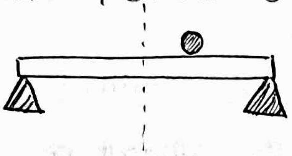
~~$T_2 = 5\times37\times10^{-3}$~~

$\boxed{T_2 = 0.31}$

$\begin{array}{r} 37 \\ \times 5 \\ \hline 0.185 \end{array}$

जितना Force उतना Torque होगा।

Ques.



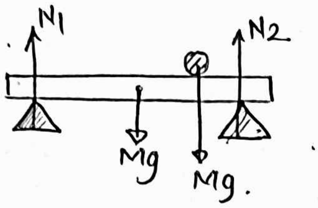
Rod length $= l$.

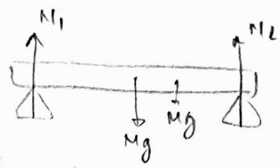
Rod का Mass $=$ stone का Mass $= M$

stone is kept at a dist $\frac{l}{4}$ from centre.

find Normal Rxn due to supports

Solu^n





Forces

$$N_1 + N_2 = 2Mg.$$

Torque

$$\tau_{N_1} = 0.$$

$$\tau_{N_2} = N_2 l \odot$$

$$\tau_{mg} = Mg\frac{l}{2} \otimes$$

$$\cancel{N_2 l} = \tau_{mg(ball)} = Mg\frac{3l}{4} \otimes$$

$$\Rightarrow \frac{3Mgl}{4} + \frac{Mgl}{2} = N_2 l.$$

$$\Rightarrow \frac{5mgl}{4} = N_2 l$$

$$\boxed{N_2 = \frac{5mg}{4}}$$

$$\boxed{N_1 = \frac{3Mg}{4}}$$

$\frac{2l}{4} + \frac{l}{4}$

$N_1 + N_2 = 2Mg.$

$\tau_{N_1} = 0$

$\tau_{N_2} = N_2 l \odot$

$\tau_{Mg} = Mg\frac{l}{2} \otimes$

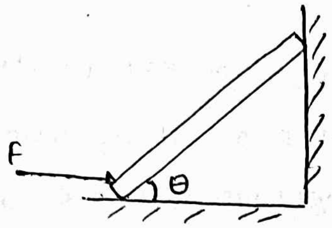
$\tau_{N_2} = Mg\frac{3l}{4} + \frac{Mgl}{2} = N_2 l$

$\frac{5Mg}{4} = N_2$

$N_1 = \frac{8Mg}{4} - \frac{5Mg}{4}$

$\Rightarrow \frac{3mg}{4}$

Ques.

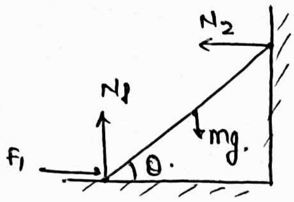


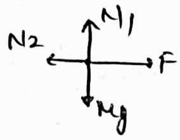
Rod का Mass = M + length = L.

All surfaces are smooth

find Mag. of Horizonatal force applied as shown to keep rod in equilibrium.

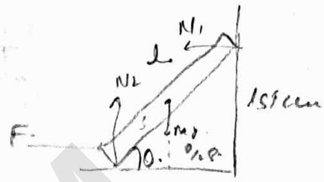
Sol.





$F = N_2$

$N_1 = Mg.$



$N_1 = F.$

$\tau_F = 0.$

$\tau_{N_2} = 0.$

$\tau_{mg} = Mg\frac{l}{2}\sin\theta$

$\tau_{N_1} = N_1 l \sin\theta$

$N_1 l \sin\theta = Mg\frac{l}{2}\frac{\sin\theta}{\cos\theta}$

$N_1 = \frac{Mg}{2}\sin$

$F = \frac{Mg\cos\theta}{2\sin\theta}$

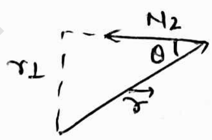
① $\tau_{mg} =$

$r_\perp = \cancel{\frac{l}{2}\sin\theta}.\ \frac{l}{2}\cos\theta.$

$\cancel{\tau_{mg}} = \cancel{mg\frac{l}{2}\sin\theta}.$

$\tau_{mg} = mg\frac{l}{2}\cos\theta$ ⊗

② $\tau_{N_2} =$



$\cancel{r_\perp = l\cos\theta}.\ r_\perp = l\sin\theta$

$\tau_{N_2} = \cancel{mg}N_2\, l\sin\theta$ ⊙

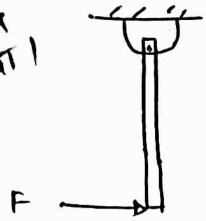
~~mgl sinθ~~

$N_2 l\sin\theta = mg\, l\cos\theta.$

$N_2 = \frac{Mg\cos\theta}{2\sin\theta}$

$\boxed{F = \frac{Mg}{2\tan\theta}}$

Ques. Test मे गलत कर दिया था।



Rod Mass = M & length = L

Horizontal Force F at bottom most point such that $F = 2Mg$. Find Angular acc. generated.

Sol.

$\tau = F r_{\perp}$ $\qquad$ $\tau = I\alpha$

$F r_{\perp} = I\alpha$

$2MgL = \frac{ML^2}{3}\alpha$

$\boxed{\alpha = \frac{6g}{L}}$

$2mgl = \frac{4ml^2}{3}\alpha$

$\frac{6g}{l} = \alpha$

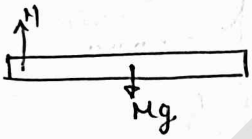
$2mgl = \frac{ml^2}{3}\alpha$

$\frac{6g}{l}$

Ques.

Find Angular acc. & Normal Rxn. just after string is cut.

Sol.



$\tau = I\alpha$

$\frac{Mgl}{2} = \frac{Ml^2}{3}\alpha$

$\frac{3g}{2l}$

$Mg - N = \frac{3mg}{4}$

* Forces

$Mg - N = ma_{cm}.$

* Torque

$\tau = I\alpha.$

$Mg\frac{L}{2} = \frac{ML^2}{3}\alpha$

$\alpha = \frac{3g}{2L}$

$a_{cm} = \alpha r.$

$a_{cm} = \alpha\frac{L}{2}.$

$a_{cm} = \frac{3g}{4}$

$Mg - \frac{3mg}{4} = N$

$\boxed{N = \frac{Mg}{4}}$

Que.

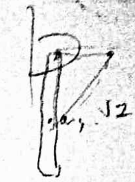
α, N when rod become vertical

$\tau = 0 = I\alpha$

$\alpha = 0$

$N - Mg = 0.$

$\boxed{N = Mg}$



$\omega = \alpha t^2$

$\omega^2 = 2\alpha\theta$

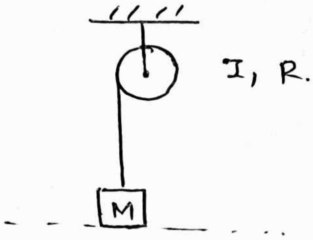
$\frac{1}{2} I\omega^2 = mgL$

$\frac{1}{2}\frac{ML^2}{3}\times 2\alpha \ldots = mgl$

$\alpha = \frac{3\sqrt{2}g}{l}$

$\tau =$

Que.



T = ?

a = ?

Sol.

Force

$Mg - T = Ma$

Torque

$TR = I\alpha$

$T = \frac{Ia}{R^2}$

$a_t = R\alpha.$

$I\times r = I\alpha.$

$\frac{MR^2}{2}\times\alpha = Mg$

$\alpha = \frac{2g}{R}$

$a_t = R\alpha$

$\frac{a_{cm}}{R} = \frac{2g}{R}$

$Mg - T = 2g.$

$Mg - \frac{Ia}{R^2} = Ma.$

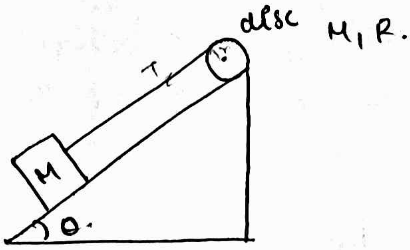
$a = \left[\frac{Mg}{\left(M + \frac{I}{R^2}\right)}\right]$

$\boxed{a = \frac{MR^2 g}{MR^2 + I}}$

$\frac{MR^2 g}{MR^2 + \frac{MR^2}{2}}$

$\frac{2MR^2 g}{3MR^2}$

Que.



$T = ?$

$a = ?$

$Mg\sin\theta - T = Ma$

$a = \frac{MR^2 g}{MR^2}$

$T =$

$\frac{2g}{3}$

$Mg\sin\theta - T = \frac{M2g}{3}$

$Mg\sin\theta - \frac{2Mg}{3} = T$

Sol.

$$Mg\sin\theta - T = Ma$$

$$TR = \frac{MR^2}{2} \times \frac{a}{R}$$

$$\boxed{T = \frac{Ma}{2}}$$

$$Mg\sin\theta = \frac{3Ma}{2}$$

$$\boxed{a = \frac{2g\sin\theta}{3}}$$

Que. A Force $\vec{F} = (3\hat{i} + 2\hat{j} + \hat{k})\,N$ is applied on a point $(2,3,4)$ Find Torque of this force about the point $(1,2,3)$

Sol.

$$\vec{r} = +\hat{i} + \hat{j} + \hat{k}$$

$$\vec{F} = 3\hat{i} + 2\hat{j} + \hat{k}$$

$$\tau = \vec{r} \times \vec{F} = (\hat{i} + \hat{j} + \hat{k}) \times (3\hat{i} + 2\hat{j} + \hat{k})$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(1-2) - \hat{j}(1-3) + \hat{k}(2-3)$$

$$\boxed{\tau = -\hat{i} + 2\hat{j} - \hat{k}}$$

18/11/19 TL-65.

ANGULAR MOMENTUM

* It is moment of linear momentum. i-e

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

$\vec{p}$ = linear momentum. of moving particle.
$\vec{r}$ = position vector w.r.t origin.

* $\vec{L} = \vec{r} \times m\vec{v}$

$= \cancel{\vec{r}}\, m(\vec{r} \times \vec{v})$

$$\boxed{\vec{L} = m[\vec{r} \times (\vec{\omega} \times \vec{r})]}$$ $\Rightarrow$ vector triple product.

* $\vec{L} \perp \vec{r} \Rightarrow \vec{L}\cdot\vec{r} = 0$
$\vec{L} \perp \vec{v} \Rightarrow \vec{L}\cdot\vec{v} = 0$
$\vec{L} \perp \vec{p} \Rightarrow \vec{L}\cdot\vec{p} = 0$

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

Magnitude

$\Rightarrow L = rp\sin\theta$

$\Rightarrow \boxed{L = mvr\sin\theta}$

$\boxed{L = mvr_{\perp}}$

Dirc^n

$\downarrow$

$\perp$ar to plane containing $\vec{r}$ & $\vec{p}$

(RHTR)

* $\boxed{\vec{L} = I\vec{\omega}}$

$\vec{\omega}$ = Axial vector
$\vec{L}$ = Axial vector

* $\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$ * $\vec{\tau}_{av} = \frac{\Delta \vec{L}}{\Delta t}$

$$\boxed{\vec{\tau}_{av} = \frac{\vec{L}_2 - \vec{L}_1}{\Delta t}}$$

* Angular impulse

$$d\vec{L} = \vec{\tau} \cdot dt$$

$$\Delta \vec{L} = \int d\vec{L} = \int \vec{\tau}\, dt$$

* Rotational Power

$$\boxed{P = \vec{\tau} \cdot \vec{\omega}}$$

* Rotational K.E

$$\boxed{K = \frac{1}{2} I \omega^2}$$

* Work Done

$$\boxed{\int dW = \int \vec{\tau} \cdot d\vec{\theta}} = \Delta K.$$

Neet 2017

Ques. $m = 2kg$ is moving along straight line ~~2x+3y=5~~ $2x + 3y + 5 = 0$. Finds its Angular momentum about origin. spd of particle $= 3m/s$

sol.

$$L = mvr_{\perp} \Longrightarrow r_{\perp} = \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \quad \frac{5}{\sqrt{13}}$$

$$L = 2 \times 3 \times \frac{5}{\sqrt{13}}$$

$$\Rightarrow \frac{0 + 0 + 5}{\sqrt{13}}.$$

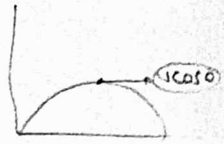
Que. A particle of mass 'm' is projected at angle 'θ' to horizontal with spd. 'u' from origin. Find Angular momentum about origin when it is at

(A) Highest point of trajectory

(B) The point when it hits the ground.

Sol.

General condition



$\vec{r} = x\hat{i} + y\hat{j}$

$\vec{v} = v_x\hat{i} + v_y\hat{j}$

$L = mvr_\perp$

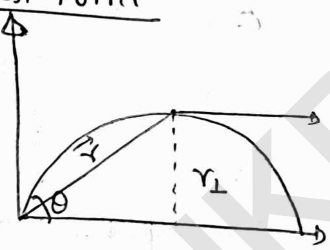
$\vec{L} = m(\vec{r} \times \vec{v})$

$= m \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & 0 \\ v_x & v_y & 0 \end{bmatrix}$

$= m(xv_y - yv_x)\hat{k}$

$$\left[\begin{array}{l} x = u\cos\theta\, t \\ y = u\sin\theta\, t - \frac{1}{2}gt^2 \\ v_x = u\cos\theta \\ v_y = u\sin\theta - gt \end{array}\right]$$

At Highest Point



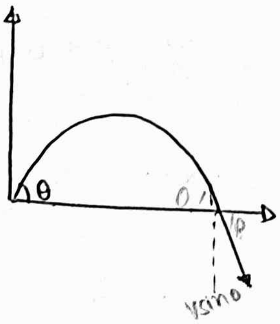
$L = mvr_\perp$

$= m\,u\cos\theta \left[\frac{u^2\sin^2\theta}{2g}\right]$

$\boxed{L \Rightarrow \frac{mu^3}{2g}\sin^2\theta\cos\theta\,(-\hat{k})}$

$\theta = 45°$

$\boxed{L = \frac{mu^3}{4\sqrt{2}g}}$



$L = mvr_\perp$

$\Rightarrow mu\sin\theta\,(R)$

$\Rightarrow mu\sin\theta\left(\frac{u^2\sin 2\theta}{g}\right)$

$\boxed{L \Rightarrow \frac{mu^3\, 2\sin^2\theta \cdot \cos\theta}{g}}\ (-\hat{k})$

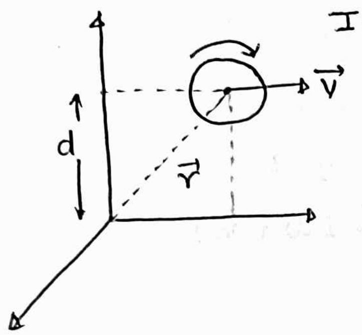
OR

$L = m(\vec{r} \times \vec{v})$

$\Rightarrow m \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ R & 0 & 0 \\ u\cos\theta & -u\sin\theta & 0 \end{bmatrix}$

$L \Rightarrow -mRu\sin\theta \, (-\hat{k})$

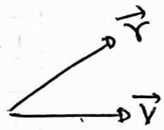
Ques



I = Given Angular velocity = ω.

$\vec{L} = ?$

Sol.



$\vec{L}_{translational} = m(\vec{r} \times \vec{v})$

$= mvr_{\perp} (-\hat{k})$

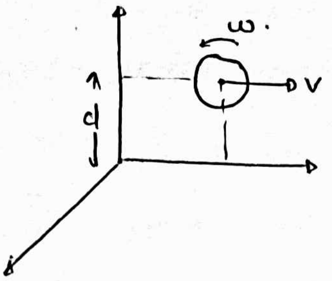
$= mvd(-\hat{k})$

$\vec{L}_{rotational} = I\vec{\omega}$

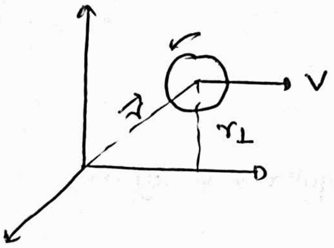
$\Rightarrow I\omega(-\hat{k})$

$\boxed{L_{Net} = (mvd + I\omega) - \hat{k}}$

Que.



Sol.



$\vec{L}_T = m(\vec{r} \times \vec{v})$

$= m v r_\perp$

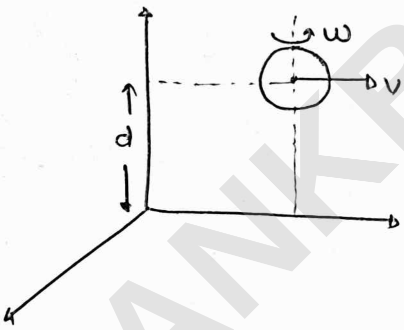
$= m v d(-\hat{k})$

$\vec{L}_R = I\vec{\omega}$

$= I\omega(\hat{k})$

$\boxed{L_{Net} = (I\omega - mvd)\hat{k}}$

Que.



coin of mass = m radius = R
rotating as shown while moving
with $V_{cm} = V$

$\vec{L} = ?$

Sol.

$\vec{L}_T = m v r_\perp$

$\Rightarrow m v d(-\hat{k})$

$\cancel{\vec{L}_R = I\vec{\omega}(\hat{j})}$

$\cancel{= \frac{MR^2}{2}\omega(\hat{j})}$

$\cancel{L_{Net} = \frac{MR^2\omega}{2}\hat{j} - Mvd\hat{k}}$

Ques. A wheel (disc) has $R = 25cm$. it's mass $= 16\ kg$. Find Torque needed to ies its Angular spd. from 0 to 120 rev./min. in 8 sec.

sol.

$$\tau = I\alpha$$

$$\Rightarrow I\frac{d\omega}{dt}$$

$$\Rightarrow \frac{16}{2} \times \left(\frac{1}{4}\right)^2 \times \frac{4\pi}{8}$$

$$\tau \Rightarrow \frac{\pi}{4}\ N\text{-}m$$

$\tau = I\alpha$

$\frac{120 \times 2\pi}{60}$

4π

$\frac{120 \times 2\pi}{60}$

$4\pi = \alpha \times 8$

$\alpha = \frac{\pi}{2}$

$\frac{MR^2}{2}$

$16 \times \frac{1}{16} \times \frac{\pi}{2}$

$\pi/2\ N\text{-}m$

CONSERVATION OF ANGULAR MOMENTUM

* Acc. to it when external Torque. is absent, then Angular momentum of a body or system remains constant. it doesn't matter if there any internal change take place.

i.e

$$\Rightarrow \vec{\tau}_{ext} = 0$$

$$\Rightarrow \frac{d\vec{L}}{dt} = 0$$

$$\Rightarrow \boxed{L = const.}$$

* $\boxed{\vec{L_i} = \vec{L_f}}$

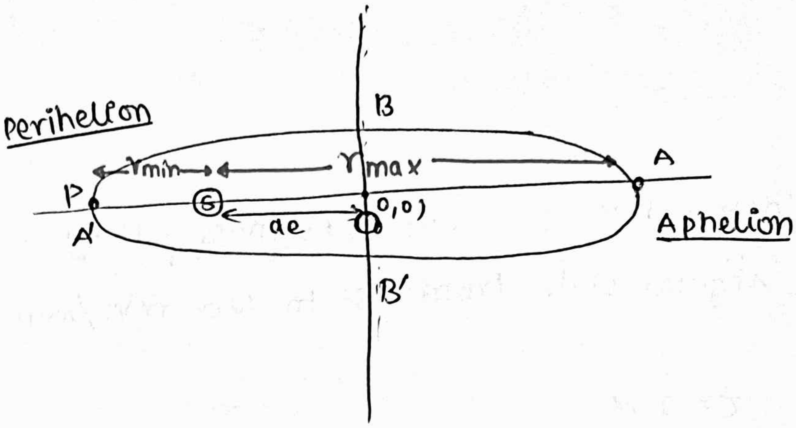
* $\boxed{m(v_1 r_1) = m\, v_2 r_2}$ $\Rightarrow$ Planetary Motion

* $\boxed{I_1\omega_1 = I_2\omega_2}$ $\Rightarrow$ Rigid body Rotation.

* Elliptical orbit में Planetary Motion

e = eccentricity

$= \sqrt{\dfrac{a^2-b^2}{a}}$



$$\boxed{\begin{aligned} r_{min} &= a - ae \\ r_{max} &= a + ae \end{aligned}}$$

$OA = OA' = a$

$OB = OB' = b$

Angular momentum conservation.

$$m(vr) = \text{const.}$$

$$\boxed{v \propto \frac{1}{r}}$$

$V_P = V_{Max}$ | $V_A = V_{min}$

$r_P = r_{min}$ | $r_a = r_{max}$.

$$\boxed{\frac{V_a}{V_P} = \frac{V_{min}}{V_{max}} = \frac{r_P}{r_a} = \frac{r_{min}}{r_{max}}}$$

$$\boxed{\frac{V_{min}}{V_{max}} = \frac{r_{min}}{r_{max}} = \frac{1-e}{1+e}}$$

★ Applications of $I_1\omega_1 = I_2\omega_2$:-

Ques. A disc of Mass M, Radius R is rotating about its Geometrical axis a particle of mass $\frac{M}{2}$ is attached gently to outer rim find new Angular spd.

Sol.

$$\frac{MR^2}{2}\omega = \left[\frac{MR^2}{2} + \left(\frac{M}{2}\right)R^2\right]\omega'$$

$$\boxed{\omega' = \frac{\omega}{2}}$$

$\frac{MR^2}{2}\omega = \frac{3MR^2}{2}\omega'$

$\omega' = \frac{\omega}{3}$

* loss in K.E :-

$$\boxed{K = \frac{1}{2}\frac{I_1 I_2}{(I_1+I_2)}(\omega_1 - \omega_2)^2}$$

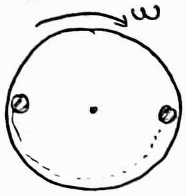
Ques. An insect is crawling on a rotating disc

i) Along circumference ii) Along diameter.

To reach at ~~the~~, exactly opp. end. Find variation in its Angular spd. of system.

Sol.

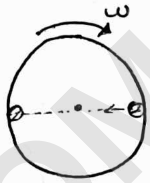
① Along circumference



$I\omega$ = const.

$\boxed{\omega = \text{const.}}$

② Along diamtr.



$(I + mR^2) = I_{initial}$

I = 1st ↓es then ↑es

(Min = insect at center

ω = First ↑ then ↓es

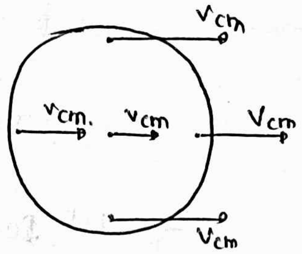
Max = insect at center

L-6

ROLLING MOTION + ENERGIES

* **Translatory K.E :-** when a body changes its position w.r.t time then it is considered to be in motion. The K.E associated with this motion is c/a. Translatory K.E i.e

$$\boxed{K_T = \frac{1}{2} m v_{cm}^2}$$



* **Rotational K.E :-** when a body performs rotational motion then energy associated with is c/a Rotational K.E i.e

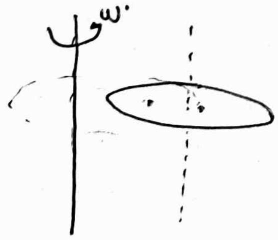
$$\boxed{K_R = \frac{1}{2} I \omega^2}$$

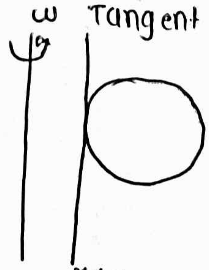
* In pure rotational motion

→ when axis of rotation is passing through the body, then every point except those point which lie on axis of rotation performs their individual circular motion.

→ All points on axis on rotation will be at rest.

* when axis of rotation is out of body, then every point of body performs the circular motion.



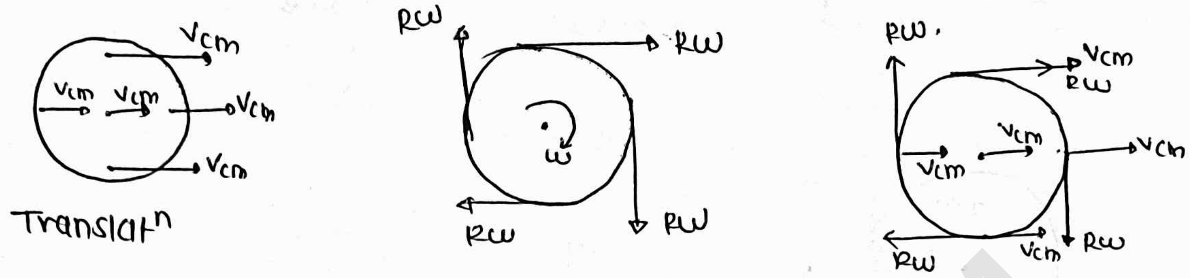


xx (Not out of body)

* Rolling motion

* Rolling + pure rolling are diff conceptually.

* it is combined motion of Translation + Rotation.



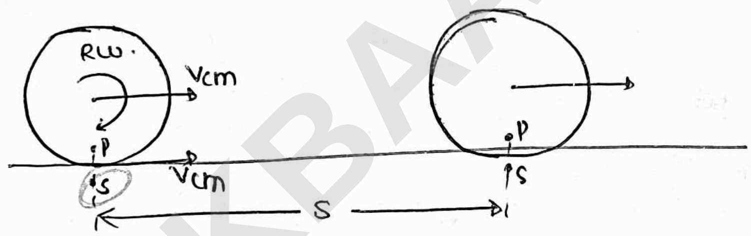
$$\boxed{\text{Total Energy} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2}$$

$V = 0$ of v_{cm} Test में गलती।

* Pure Rolling

- it is special case of Rolling.

* condition for pure rolling.



$S = V_{cm}$ (Potion)

$2\pi R = R\omega$.

*

$S = 2\pi R$	Pure rolling
$S > 2\pi R$	forward slipping
$S < 2\pi R$	Backward slipping.

* in pure rolling → Rel. velocity of contact point $= 0$.

* $\vec{V}_{PS} = 0 \quad \vec{V}_P = \vec{V}_S$ (w.r.t ground)

$\vec{V}_{cm} - R\omega = 0$

Pure rolling $\boxed{a_{cm} = \alpha R}$

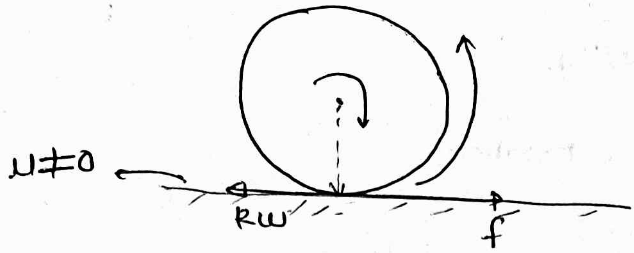
$V_{cm} > R\omega$

Forward slipping

$V_{cm} < R\omega$

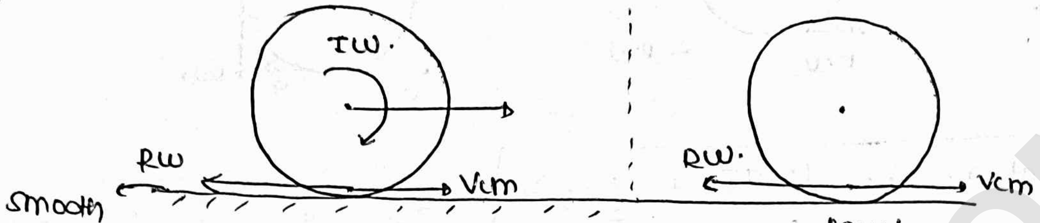
Backward slipping.

*



Rotating Body को Rough surface पर रख दो तो 'f' के कारण $\omega \downarrow$

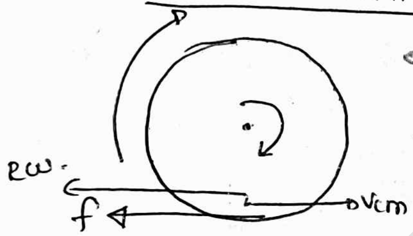
*



Rough.

$V_{cm} > R\omega$

FORWARD SLIPPING



जब car spd. है और suddenly Brake Apply कर दे तो Tyre skid करेगे + will not rotate. so frictn backward

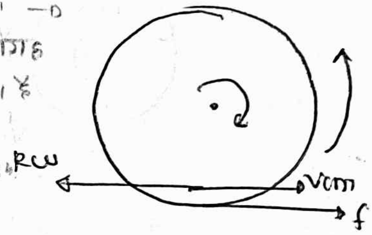
जब car रेत में हो → तो Tyre एक जगह घूमने लग जाता है, only rotate + no translational motn Rw only

$\omega \uparrow$ कब तक =?

जब तक की Pure Rolling start ना हो

$V_{cm} < R\omega$

BACKWARD SLIPPING

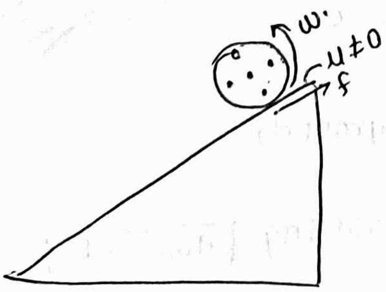


$\omega \downarrow$ कब तक =?

जब तक Pure rolling start ना हो।

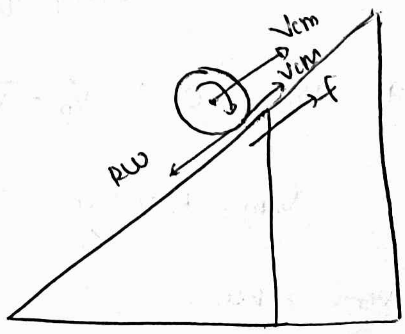
* Rolling motion on inclined surface

Ⓐ Down the incline



Friction acts up the incline

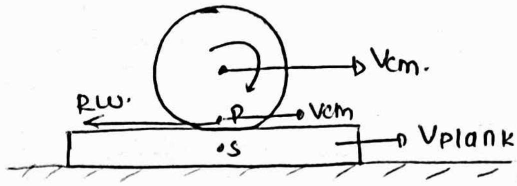
Ⓑ Up the incline



Fricn up the incline

★ Rolling on moving surface

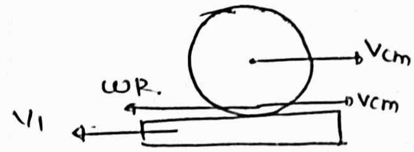
*



$$\vec{V_p} = \vec{V_s} \Rightarrow \vec{V}_{plank}.$$

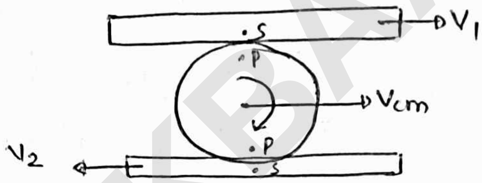
$$\boxed{V_{cm} - \omega R = V_{Plank}}$$

*



$$\boxed{\omega R - V_{cm} = V_1}$$

*

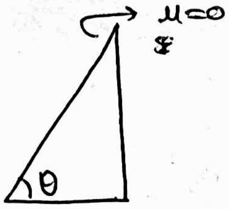


$$\boxed{\begin{aligned} V_1 &= V_{cm} + \omega R \\ V_2 &= R\omega - V_{cm} \end{aligned}}$$

Energies of Diff bodies in pure rolling :-

~~Energy~~ Body	I	K	K^2/R^2	K_T	K_R	T.E
i) Ring (Hollow cyl.)	MR^2	R	1	$\frac{1}{2}mv^2$	$\frac{1}{2}mv^2$	Mv^2
ii) Disc (solid cyl.)	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{1}{2}mv^2$	$\frac{Mv^2}{4}$	$\frac{3}{4}mv^2$
iii) shell	$\frac{2}{3}MR^2$	$R\frac{\sqrt{2}}{\sqrt{3}}$	$\frac{2}{3}$	$\frac{1}{2}mv^2$	$\frac{1}{3}Mv^2$	$\frac{5}{6}mv^2$
iv) sphere	$\frac{2}{5}MR^2$	$\frac{R\sqrt{2}}{\sqrt{5}}$	$\frac{2}{5}$	$\frac{1}{2}mv^2$	$\frac{mv^2}{5}$	$\frac{7}{10}mv^2$

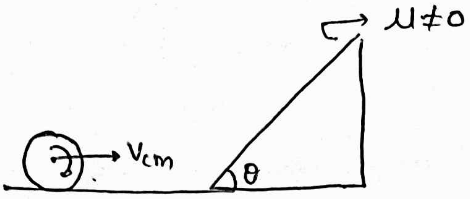
Ques.



Find Height attained by body?

Sol. For each body $\boxed{h = \frac{v^2}{2g}}$ $\Rightarrow$ B'coz surface is smooth.

Ques.



$Ht = ?$

$K_T = mgh.$

Ring $\rightarrow$ $h = v^2/2g$

PYQ

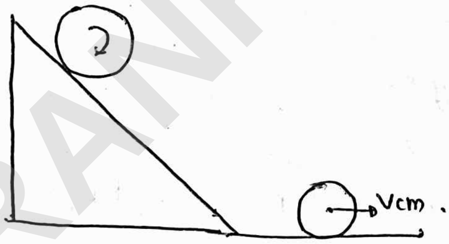
disc $\rightarrow$ $mgh = \frac{3}{4}mv^2$

$h = \frac{3}{4}\frac{v^2}{g}$

shell $\rightarrow$ $h = \frac{5}{6}\frac{v^2}{g}$

sphere $\rightarrow$ $h = \frac{7}{10}\frac{v^2}{g}$

Ques.



$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

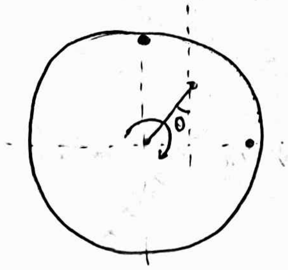
Ring $v = \sqrt{2gh}$

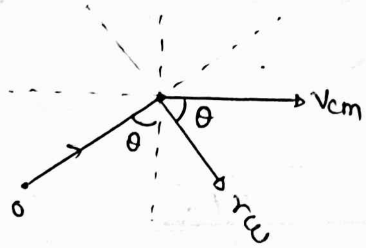
Disc $v = \sqrt{\frac{4}{3}gh}$

shell $v = \sqrt{\frac{6}{5}gh}$

sphere $v = \sqrt{\frac{10gh}{7}}$

Velocity of Any point on body performing rolling :-



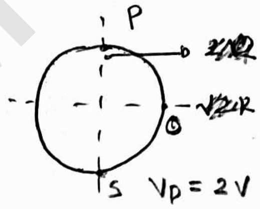


θ = is with vertical.

$$\boxed{V_P = \sqrt{(V_{cm})^2 + (\omega r)^2 + 2(V_{cm})(\omega R)\cos\theta}}$$

* Pure rolling $\boxed{V_{cm} = R\omega}$

$$\boxed{V_P = \omega\sqrt{R^2 + r^2 + 2Rr\cos\theta}}$$



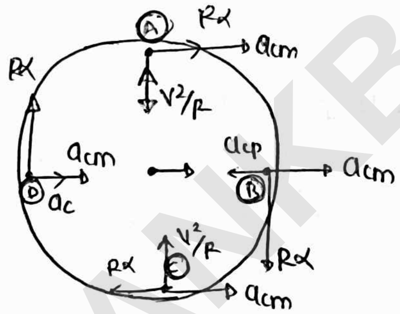
$V_P = 2V$

$V_O = V$

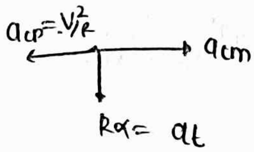
$V_S = 0.$

ACC of any point on the body :-

* Points of rim



$a_B =$



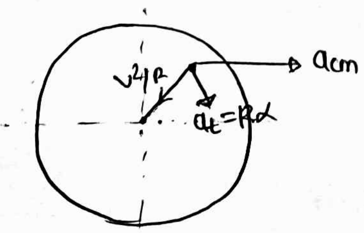
$$\vec{a}_B = \left[a_{cm} - \frac{V^2}{R}\right]\hat{i} + \alpha R(-\hat{j})$$

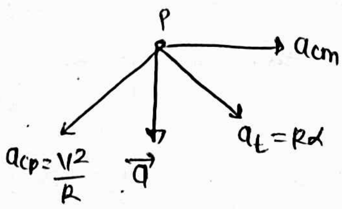
$$\vec{a}_A = [a_{cm} + R\alpha]\hat{i} - \frac{V^2}{R}\hat{j}$$

$$\vec{a}_C = [a_{cm} - R\alpha]\hat{i} + \frac{V^2}{R}\hat{j}$$

$$\vec{a}_D = \left[a_{cm} + \frac{V^2}{R}\right]\hat{i} + R\alpha\hat{j}$$

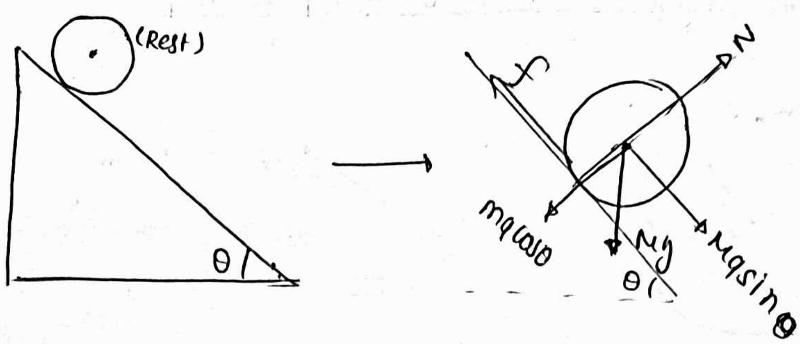
For a general point :-





$\vec{a}_P$ = Resultant of $\vec{a}_{cm}$, $\vec{a}_{cp}$ & $\vec{a}_t$

When a body Rolls down the incline :-



Force equ. = $Mg\sin\theta - f = ma_{cm}$.

$\boxed{a_{cm} = R\alpha}$

Torque equ. = $fR = I\alpha$.

$$\boxed{f = \frac{a_{cm} I}{R^2}}$$

$$a_{cm} = \frac{Mg\sin\theta}{m + \frac{I}{R^2}} = \frac{mg\sin\theta}{m + m\frac{K^2}{R^2}}$$

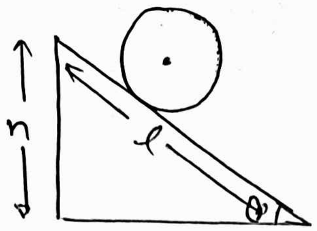
$\boxed{I = mK^2}$

$$\boxed{a_{cm} = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}}$$

जिसके लिए $\frac{K^2}{R^2}$ more ⇒ a_{cm} less

$$\boxed{f = \frac{Mg\sin\theta\,(K^2/R^2)}{\left[1 + \frac{K^2}{R^2}\right]}}$$

* <u>which body will reach earlier?</u>



$\sin\theta = \frac{h}{l} \Rightarrow \boxed{l = \frac{h}{\sin\theta}}$

$l = \frac{1}{2}at^2$

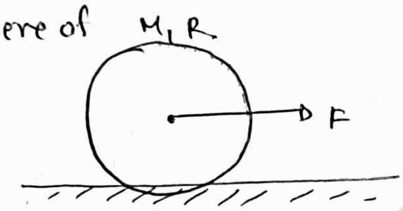
$t = \sqrt{\frac{2l}{a}}$

$$\boxed{t = \frac{1}{\sin\theta}\sqrt{\frac{2h\left(1+K^2/R^2\right)}{g}}}$$

$\boxed{T \propto \frac{K^2}{R^2}}$

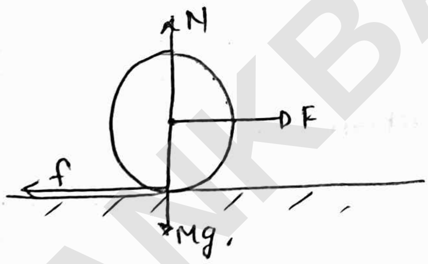
time ज्यादा $\Rightarrow$ जिसका $\frac{K^2}{R^2}$ more.

Que. sphere of M, R



Max. value of horizontal force F for pure rolling ?

Sol.



Force eq:- $F - f = ma$

Torque eq:- $fR = I\alpha$

$f = \frac{Ia}{R} = \frac{\frac{2}{5}MR^2 a}{R^2}$

$f \Rightarrow \frac{2ma}{5}$

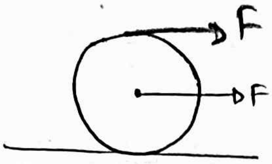
$a_{max} = \frac{5}{2m}\mu mg = \frac{5\mu g}{2}$

$F_{max} = f_{max} + ma_{max}.$

$= \mu Mg + \frac{5\mu mg}{2}$

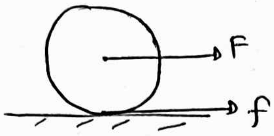
$$\boxed{F_{max} \Rightarrow \frac{7\mu mg}{2}}$$

Ques. Jee mains.



$F = ?$ जिससे pure rolling (cont.) किया जा सकेगा

Sol.



$$F + f = ma$$

$$fR = I\alpha$$

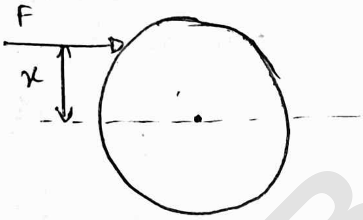
$$f = \frac{2ma}{5}$$

$$a_{max} = \frac{5\mu g}{2}$$

$$F = \frac{3ma}{5} \qquad F_{max} = \frac{3\mu mg}{5}$$

Ques.

#



1) $K^2 = Rx$ No friction

2) $K^2 > Rx$ $\longleftarrow$

3) $K^2 < Rx$ $\longrightarrow$

* Torque $Fx = I\alpha$

$$Fx = I\left[\frac{a_{cm}}{R}\right]$$

$$\boxed{F = \frac{MK^2 a_{cm}}{Rx}} \Rightarrow \text{Pure Rolling.}$$

$$\Downarrow$$

$$F_{ext} = ma_{cm}$$

$$\alpha = \frac{a_{cm}}{R}$$

when pure Rolling is not ~~taking~~ occuring.

$$a_p = a_{cm} - R\alpha$$

$$a_p = a_{cm} - R\left[\frac{Fx}{I}\right]$$

$$a_p = a_{cm} - \frac{M a_{cm}}{MR^2}[Rx]$$

$$a_p = a_{cm} - \left[1 - \frac{Rx}{k^2}\right]$$

$a_p = +ve = F.S \qquad k^2 > Rx$

$a_p = -ve = f \rightarrow \boxed{k^2 < Rx}$