UNIT AND DIMENSIONS (TL-5)

* Physical Quantity → Those quantities in terms of which we define laws of physics.

ex. spd., dist., energy etc

* Measurements

To measure a Physical Quantity we need $\boxed{\text{Magnitude} = \text{Numerical value} + \text{units}}$

* Systems of Unit.

• MKS • CGS • FPS

* SI

① Base Quantities.

* Fundamental Quantities.

• ~~Mass~~ Those quantities which do not depend on other physical quantities for their definition

• Seven fundamental quantity are listed :-

①	Mass	Kilogram	(kg)
②	Length	Metre	(m)
③	Time	second	(s)
④	Temperature	Kelvin	(K)
⑤	Electric current	Ampere	(A)
⑥	Amount of subs.	Mole	(mol.)
⑦	Luminous intensity	candela	(cd.)

② Derived Quantities.

- every other quantity which is not fundamental quantity.
- Those quantities which depend on fundamental quantities for their definitⁿ.

eg:- Force, Energy, Momentum etc.

★ Types of units.

① Fundamental units :- Units of fundamental quantities.

② Derived units :- units of derived Quantities.

eg :- Newton, Joule etc

③ supplementary units :-

i) Plane Angle → unit = radian (rad.)

~~Arc/length~~ arc/rad.

ii) Solid Angle → Angle subtended on centre of a sphere by a spherical Arc.

$$= \frac{\text{Area of spherical Arc}}{(\text{Radius})^2}$$

$$= \frac{\Delta S}{r^2}$$

unit → Steradian (st)

④ Practical units :-

*Energy

→ Joule
→ calorie . 1 cal = 4.2 J
→ eV 1 ev = 1.6×10^{-19} J
→ KwH 1 KwH = 3.6×10^{6} J.

* Power.

→ watt
→ Horse power

* length

→ metre / cm / inch / km
→ fermi (10^{-15})
→ Micron (10^{-6})

* light Year
* AU

⑤ Improper units

Those units which are used to define diff. Physical quantities other than the quantity for which they meant to be

eg:- Kg - wt = force   10N   module 6

light - year = Distance

7/07   L-2   TL-12

DIMENSIONAL ANALYSIS

* Dimension → Powers of fundamental quantities.

Force → $[M^1 L^1 T^{-2}]$

Dimensions of MLT → 1, 1, -2.

$\underbrace{[F] = [M^1 L^1 T^{-2}]}_{\text{Dimensional eq.}}$ → Dimensional formula.

Dimensional formula.

[M L T A K mol cd]

Q. Density = $\frac{\text{Mass}}{\text{Vol.}}$ = $[D] = [M^1 L^{-3} T^0]$

NOTE :-

① Any quantity which can have units must have dimensions. [not necessary have every time].

② Dimension less Quantities can have units

unitless quantities are dimensionless too.

③ Foll. quantities are dimensionless:-

- Plane angle
- solid angle
- Relative density
- Refractive index
- coeff. of friction
- strain
- Relative permeability (μ_r)
- ϵ_r.

- All trignometric, exponential, log function are Dimensionless functions.

(4) const may or may not have dimensions.

(5) <u>foll. quantities have same dimensions.</u>

$\frac{GMM}{r}$ M^2L^{-1}

(a) Planck's const and Angular momentum.

(b) Acc. due gravity, gravitational field intensity, acc.

(c) Force, weight, Thrust and energy gradient.

(d) Ryedberg const., Propogation const., wave no.

(e) Boltman const., Gas const., Entropy

(f) Momentum, Impulse

(g) Surface Tension, spring const.

(h) Gravitational potential, square of velocity, $[\mu_0 \epsilon_0]^{-1} = c^2$

* <u>Energy</u> = $[M^1L^2T^{-2}]$
  - K.E
  - P.E ⟹ work = Torque.
  - N.E
  - Heat

* <u>Power</u> = $[M^1L^2T^{-3}]$

= Mech. Power

= Electrical Power

= Thermal ~~Power~~ current ⟶ $\frac{dQ}{dt}$ = rate of flow of Heat.

= Emissive Power
 $(e = \sigma AT^4)$

* <u>Field intensity</u>

(a) Gravitational field intensity → $[L^1T^{-2}]$ $E = \frac{KQ}{r}$

(b) Electric field intensity ⟹ $[M^1L^1T^{-3}A^{-1}]$ M^1

(c) Mag. Field → B = $\frac{M^1L^1T^{-2}}{[AT][LT^{-1}]} = [M^1T^{-2}A^{-1}]$ $F = qvB$ $\frac{F}{B} = c$

$\frac{M^1L^1T^{-2}}{L^1L^{-1}}$

* ε_0

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = \varepsilon_0 = \frac{A^2T^2}{[M^1L^1T^{-2}][L^2]} = [M^{-1}L^{-3}T^4A^2]$$

* μ_0

$$\Rightarrow \quad C^2 = \frac{1}{\mu_0\varepsilon_0} \Rightarrow \mu_0 = \frac{1}{C^2\varepsilon_0} = \frac{1}{[L^2T^{-2}][M^{-1}L^{-3}T^4A^2]} = \frac{\cancel{M^1}\cancel{L^1}\ 1}{M^{-1}L^{-1}T^{+2}A^2}$$

$$= [\cancel{M^1}\cancel{L^1}\cancel{T^{-2}}A^{-2}]$$

$$= \frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{I^2}{d} . \qquad \mu_0 = \frac{M^1L^1T^{-2}}{A^2T^2} = [M^1L^1T^{-2}A^{-2}]$$

* η

$$F = 6\pi\eta v r \qquad \frac{M^1\cancel{L}T^{-2}}{L^1T^{-1}\times\cancel{L^1}} = [M^1L^{-1}T^{-3}]$$

Thermo.

* Heat $= [M^1L^2T^{-2}]$ = Energy.

* Latent Heat $=$

$Q = mL$ $\qquad L = \frac{Q}{m} = \frac{M^1L^2T^{-2}}{M} = [L^{+2}T^{-2}]$

* Specific Heat.

$Q = ms\Delta\theta$ $\qquad s = \frac{Q}{m\Delta\theta} = \frac{M^1L^2T^{-2}}{[M^1][K]} = [L^2T^{-2}K^{-1}]$

* Thermal current

$$i = \frac{dQ}{dt} = \frac{KA\Delta\theta}{L} =$$

$[i] = [M^1L^2T^{-3}]$

* Thermal conductivity (K) $\quad K = \frac{iL}{A\Delta\theta} = \frac{[M^1\cancel{L^2}T^{-3}][L]}{[\cancel{L^2}][\theta]} = \cancel{M^1T}[M^1L^1T^{-3}\theta^{-1}]$

* Thermal Resistance

$$R_{th} = \frac{L}{KA} \qquad i_{th} = \frac{\Delta\theta}{R_{th}}$$

$$R_{th} = \frac{\Delta\theta}{i_{th}} = \frac{[\theta^1]}{[M^1L^2T^{-3}]} = [M^1L^2T^{-3}\theta^{+1}]$$

$F = qE$

* Potential

$= \quad V = E.r.$

$V = \frac{F r}{q}$

$= [M^1 L^2 T^{-2}][A^{-1} T^{-1}]$

$[V] = [M^1 L^2 T^{-3} A^{-1}]$

* Resistanc

$V = IR$

$R = [M^1 L^2 T^{-3} A^{2}]$

* Resistivity

$\cancel{R} \propto \frac{l}{\cancel{A}} \quad \cancel{}$

$\rho = \frac{RA}{l} = [M^1 L^3 T^{-3} A^{-2}]$

* conductivity

$\sigma = \frac{1}{\rho} = [M^{-1} L^{-3} T^3 A^2]$

* Energy density $= \frac{E}{V} = \frac{M^1 L^2 T^{-2}}{L^3} = [M^1 L^{-1} T^{-2}]$

APPLICATION OF DIMENSIONAL ANALYSIS.

1. conversion of units of a physical quantity. from one unit system to another.
2. To check accuracy of a formula. or an equation.
3. Derivation of formula.

1. CONVERSION OF UNITS.

* Magnitude must be same.

$\boxed{N_1 U_1 = N_2 U_2}$ $\quad \boxed{N \propto \frac{1}{U}}$

$\boxed{1m = 100 cm}$

Large unit small Magnitude & vice versa.

Q.1 $F = 1N$

$F =$ How many dynes.

$$N_1 U_1 = N_2 U_2.$$

$$1\,N = N_2 \text{ dyne}.$$

$$1\left[\frac{kg\,m}{s^2}\right] = N_2\left[\frac{g\,cm}{s^2}\right]$$

$$1 \times \left[\frac{10^3 g \times 10^2 cm}{s^2}\right] = N_2\left[\frac{g \times m}{s^2}\right]$$

$$N_2 = 10^5$$

Q.2 54 km/hr $\longrightarrow$? m/s

$$N_1 U_1 = N_2 U_2$$

$$54 \times \frac{10^3 \cancel{m}}{3600 \cancel{sec}} = N_2 \times \frac{\cancel{m}}{\cancel{sec}}$$

$$N_2 = \frac{54 \times 10^3}{3600}$$

$$\boxed{N_2 = 15 m/s}$$

PQM

Q. In a unit sys. units of M, L, T are 10kg, 100m and 10s and unit of force is Newton then 1 Newton is equal to how many x?.

★

$$N_1 U_1 = N_2 U_2.$$

$$= N_2\,N$$

$$= N_2 \frac{kg\,m}{s^2}$$

$$\frac{10 \times 10^3 \times 100 \times 10^2}{s^2} = N_2 \cancel{\frac{kg\,m}{sec}}$$

$$\boxed{N_2 = 10^8}$$

Mistake

$N_1 U_1 = N_2 U_2$

$1 N =$

SI	Notwen
N	1 Notwen.
Kg	x
m	y
s	z

$1x = 10\,Kg$, $1y = 100\,m$, $1z = 10\,sec$.

$$N_1 \left[\frac{Kg\,m}{s^2}\right] = N_2 \left[\frac{xy}{z^2}\right]$$

$$\cancel{N_1}\; x \left[\frac{Kg\,m}{s^2}\right] = \cancel{N_2}\,1 \times \frac{\cancel{10} \times 100}{\cancel{100}} \qquad \boxed{N_1 = 10}$$

Test mis

Q. In a unit syst. units of M, L, T are α Kg, β m, γ sec respect. what is the unit of Energy in this system.

$M' \, T$

$E = ML^2T^{-2}$

SI	New sys.
Kg	$\cancel{\alpha}$ x
m	β y
s	$\cancel{\gamma}$ z.

$1x = \alpha$, $1y = \beta$, $1z = \gamma$

$$N_1 U_1 = N_2 U_2.$$

$$N_1 \times \left[\frac{Kg\,m^2}{s^2}\right] = 1 \left[\frac{x\,y^2}{z^2}\right]$$

$$N_1 \times \frac{Kg\,m^2}{s^2} = 1\,\frac{\alpha\,\beta^2}{\gamma^2}$$

$$\boxed{N = \frac{\alpha\beta^2}{\gamma^2}}$$

A*

$$N_2 = N_1 \left[\frac{U_1}{U_2}\right]$$

$$\boxed{N_2 = N_1 \left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]\left[\frac{T_1}{T_2}\right]}$$

② CHECK THE ACCURACY OF FORMULA.

i) dimensionally correct equation may or may not be correct actually.

ii) Dimensionally incorrect equation is incorrect Always.

- $\boxed{S_{nth} = u + \frac{a}{2}(2n-1)}$ → this is eq. is dimesionally incorrect but we treat it as dimensionally correct.

iii) for the accuracy, we need principle of Homogeneity

$$\boxed{\text{Dimensions of } \quad LHS = RHS}$$

Rules

ⓐ Two quantities can be added or substracted only when, the dimensions of same quantities

ⓑ Division + Product → Quantities of diff. Dimension are allowed.

ⓒ $\boxed{e^{AB}, \; e^{A/B}, \; \ln\left(\frac{A}{B}\right) \; . \; \sin(AB)}$ ⇒ Must be dimensionless.

ⓓ e^{A+B}, $\sin(A+B)$ → A + B both must be dimensionless.

eg:- $V = V_0 e^{-Kt^2}$ $\qquad$ $V = V_0 \exp(-Kt^2)$

$K = [M^0 L^0 T^{-2}]$ $\qquad$ $[K = M^0 L^0 T^{-2}]$

Date: 11/6 $\qquad$ L-3 $\qquad$ TL-13

③ DERIVATION OF FORMULA

i) we can't derive the formula in which log, exponential, trignometric function are involved,

ii) we can't derive formula in which addition + substraction of 2 & more. terms are involved

iii) we can derive formula in which product & division of Physical quantities are involved but we can't get any information about the dimensionless const. involved.

- $s = ut + \frac{1}{2}at^2$ • $q = q_0 e^{-t/RC}$ • $x = A\sin(\omega t)$ •

$\underbrace{\qquad\qquad\qquad\qquad}_{\text{Not allowed.}}$

$T = 2\pi\sqrt{\frac{l}{g}}$ ✓

Q. expression for time period for a simple pendulum which depends on length (l), & acc due to gravity (g)

$$T \propto l^{a}$$
$$\propto g^{b}$$

Sol.

$$[M^0L^0T^1] = [M^0L^1T^0]^a [M^0L^1T^{-2}]^b$$

$$a+b=0$$
$$-2b=1$$
$$\boxed{b=-1/2}$$
$$\boxed{a=1/2}$$

$$T \propto l^{1/2} g^{-1/2}$$
$$\boxed{T \propto \sqrt{\frac{l}{g}}}$$

Q.

$$T \quad \cancel{\propto l^a} \quad \cancel{\propto \mu^b}$$

μ = linear mass density.

$$\cancel{[M^1L^1T^{-2}] = [M^0L^1T^0]^a [M^1L^{-1}T^0]^b}$$

$$\boxed{\cancel{1=b}}$$

$$v \propto T \; l \; \mu$$

$$[M^0L^0T^{-1}] = [M^1L^1T^{-2}]^a [M^0L^1T^0]^b [M^1L^{-1}T^0]^c$$

$$\boxed{0 = a+c}$$

$$\boxed{0 = a+b-c}$$

$$-1 = -2a$$

$$a = 1/2$$

$$-1 = -2a$$
$$\boxed{a = 1/2}$$

$$\boxed{c = -1/2}$$

$$0 = \frac{1}{2} + \frac{1}{2} + b$$
$$b = -1$$

$$v \propto T^{1/2} l^{-1}$$

$$\boxed{v = \frac{1}{l}\sqrt{\frac{T}{\mu}}}$$

Q. In a unit system force, velocity, time are taken as fundamental quantities. find the dimension formula for mass in this system.

Sol.

$$[M] = F^a v^b T^c.$$

$$[M^1L^0T^0] = [M^1L^1T^{-2}]^a [M^0L^1T^{-1}]^b [M^0L^0T^1]^c$$

$$a = 1$$

$$0 = a + b$$

$$b = -1$$

$$0 = -2a - b + c$$

$$0 = -2 + 1 + c$$

$$c = 1.$$

$$M = F^1 V^{-1} T^1$$

Q. A particle is performing SHM along axis of a fixed range, due to gravitational force its displacement any instant is given by $\boxed{x = A\sin\omega t}$ in this eq quantity ω found to depend on Gravitational const. (G), Mass of ring (m) & radius of ring (r) derive expression.

$$F = \frac{GMm}{r^2}$$

$$\frac{M^1L^3T^{-2}}{M^2}$$

$$M^{-1}L^3T^{-2} = G$$

$$\omega = \frac{2\pi}{T}$$

$$\omega \propto G^a m^b r^c$$

$$[M^0L^0T^{-1}] = [M^{-1}L^3T^{-2}]^a [M^1L^0T^0]^b [M^0L^1T^0]^c$$

$$0 = -a + b$$

$$\boxed{a = b}$$

$$0 = 3a + c$$

$$c = -3a$$

$$c = -\frac{3}{2}$$

$$-1 = -2a$$

$$a = 1/2$$

$$b = 1/2$$

$$\omega = G^{1/2} m^{1/2} c^{-3/2}$$

$$\omega = \sqrt{\frac{G}{M}} \frac{1}{\sqrt[3/2]{C}} \propto \sqrt{\frac{G}{MR^3}}$$

SIGNIFICANT FIGURES :-

i) Are used for the observation of a physical experiment, which can be used further in calculations.

ii) In every calculation there are some reliable digit + one unreliable digit is tnt.

iii) These all are considered as significant figures.

* Rules

1. All Non-Zero digit are significant
2. A zero b/w two non zero digit is significant
3. Trailing zero (zero to the Right of Last non-zero digit) are not significant in number without decimal.

→ if No. comes from actual Measurement then trailing zero are significant

4. → Trailing zero in a no. having decimal point are significant

eg:-

270000 = 2SF

270000 Kg = 6SF

270.000 = 6SF

4. zeroes left to the 1st non zero digits are not significant in a no. having decimal point or a no. without decimal point

eg:-

001083 = 4SF

0.0001083 = 4SF

5. When a no. is expressed in powers of 10, the no. of significant. figures remains same.

$\underline{123 = 3SF}$ → 12.3×10^1 (3) 1.23×10^2 (3) 0.123×10^3 (3)

SCIENTIFIC NOTATION.

~~a x b~~

$$a \times 10^{b}$$

$$\boxed{1 \le a < 10}$$

ORDER OF MAGNITUDE

$$a \times 10^{b}$$

OOM → b

$$\boxed{0.5 \le b < 5}$$

* Atomic radius

$= 0.529\,A^\circ$

$S.N = 0.529 \times 10^{-10}$

$= 5.29 \times 10^{-11}\,m.$

$OOM = \boxed{b = -10}$

Q. $G = 6.67 \times 10^{-11}$

$S.N = 0.667 \times 10^{-10}.$

$OOM \doteq b = -11.$

ROUNDING OFF.

→ it is process to represent result of any calculation containing more than 1 uncertain digit, to reduce the no. to the appropriate ~~fig~~. of significant fig. is rounding off.

uncertain digit
↓
last one → >5, <5, =5

- If last digit is more them 5 than preceeding digit is increased by 1.

eg:- 7.776 → 4 SF
↓ 3SF
$\boxed{7.78}$

(2) if last digit is less than 5 the preceeding digit ~~is~~ ~~are~~ remain same

eg:- 7.774 ⟶ 7.77

iii) last digit equal to 5 then precedding digit is ~~fesed~~ by.

(1) if odd

if even remain unchanged.

eg:- 7.745 = 7.74

7.775 = 7.78

Q. convert 25.653 to 3 digits.

⟶ 25.7

Q. 25.653 ⟶ 3sf.

(5) ⟶ (4) ⟶ (3)

25.653 ⟶ 25.65 ⟶ 25.6 ✗ wrong method.

Q. 4.996 ⟶ 3 digit.

~~4.99~~ 5.00

Q. 0.6995 ⟶ 1 digit

★ $\boxed{0.7}$

500

Q 11.1251 (4 digit)

⟶ 11.13

* Rules for Arithmetic operations

(1) Addition + subtraction

- In final result the no. of decimal places must be equal to, the no. of decimal places of that term in operation which contain lesser no. of decimal places

eg:- 12.587 - 12.5

$$\begin{array}{r} 12.587 \\ 12.500 \\ \hline 0.087 \\ \hline \end{array}$$

0.087 → compare with 50

↓

$\boxed{0.1}$

(2) Multiplication + Division

No. of SF in result is equal to No. of SF in the any of factor which contain smaller no. of SF.

eg:- 5.0×0.125

$= 0.625$

$= 0.62$

12/07 L-4 TL-14

ERROR ANALYSIS:-

* It is diff. b/w true value + measured value of a physical quantity.

$$\boxed{\text{Error} = \text{True Value} - \text{Measured value}}$$

* n observations

Reading ⇒ $a_1, a_2, a_3 \ldots + a_n$

* True value given in Q.

↓

a_t

* If not given

Mean value $a_m = \dfrac{a_1 + a_2 + a_3 \ldots + a_n}{n}$

we can use $\boxed{a_m = a_t}$

① ABSOLUTE ERROR :-
(Δa)

* In every observation
$\Delta a_1 = a_m - a_1$
$\Delta a_2 = a_m - a_2$
$\boxed{\Delta a_n = a_m - a_n}$ a_n = reading of nth observation.

② Mean Absolute Error

→ Avg. of all absolute error
→ defined for final reading
→ $$\boxed{\Delta a_m = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| \dots + |\Delta a_n|}{n}}$$

* Quantity (A)

Representation (A) ⇒ $\boxed{A \pm \Delta A}$ it is not a range.
↓
"within proper error limit"

③ Fractional error

* $\frac{\Delta a}{a}$ or $\frac{\Delta a_m}{a_m}$

④ % Error

$$P.E = \frac{\Delta a}{a} \times 100$$

$\boxed{\% \text{ error} = (\text{Fractional error}) \times 100}$

* CHANGE :-

= final − initial

i) change = +ve

- gain
- increase
- final > initial

② change = -ve

- loss
- $\downarrow$es
- Loss in quantity = initial - final

* $\boxed{\text{Fractional change} = \dfrac{\text{final} - \text{initial}}{\text{initial}}}$

* $\boxed{\text{\% change} = \dfrac{\text{Final} - \text{initial}}{\text{initial}} \times 100}$

* $\boxed{\text{\% Loss} = \dfrac{\text{initial} - \text{final}}{\text{initial}} \times 100}$

* $\boxed{\text{\% increase} = \dfrac{\text{Final} - \text{initial}}{\text{initial}} \times 100}$

* Fractional change in error. $\left(\frac{\Delta x}{x}\right)$.

i) for smaller changes

- use differentiation

ⓐ $y = x^n$

$$\boxed{\frac{\Delta y}{y} = n\left(\frac{\Delta x}{x}\right)}$$

ⓑ $y = \dfrac{x^n z^m}{A^p}$

$$\frac{\Delta y}{y} = n\left(\frac{\Delta x}{x}\right) + m\left(\frac{\Delta z}{z}\right) - p\left(\frac{\Delta A}{A}\right)$$

ⓒ $y = \sin x$

$$\boxed{\frac{\Delta y}{y} = \cos x\left(\frac{\Delta x}{x}\right)}$$

ⓓ $y = e^x$

$$\boxed{\frac{\Delta y}{dy} = e^x \cdot \frac{\Delta x}{x}}$$

ii) Bigger change

- more than 5%

* $$\boxed{\frac{\Delta x}{x} = \frac{x_f - x_i}{x_i}}$$

Ques. K.E of a particle is ↑es by 2%, initially it was 100 J. calculate final K.E

Soln.

$$K_i = 100\,J$$

$$K_f = K_i + \frac{x}{100} \times K_i$$

$$K_f = 100 + \frac{2}{100} \times 100$$

$$K_f = 100 + 2$$

$$\boxed{K_f = 102\,J}$$

Ques. $K = \frac{1}{2}mv^2$ $\quad$ $K = f(v)$

i) vel. is ↑esed by 2%

$$v_f = v_i + \frac{2}{100} v_i$$

$$\boxed{\frac{\Delta K}{K} = 2\frac{\Delta v}{v}}$$

2x

ii vel. is ↑es. by 20%.

$v_i = v$ $\quad$ $K_i = \frac{1}{2}mv^2$

$$v_f = v_i + \frac{20}{100} v_i$$

$$v_f = \frac{6v_i}{5}$$

$$K_f = \frac{1}{2}m\left(\frac{6v}{5}\right)^2 = \frac{1}{2}m\frac{36v^2}{25} = \frac{36mv^2}{50}.$$

$$\frac{\Delta K}{K} = \frac{K_f - K_i}{K_i} = \frac{\frac{36mv^2}{50} - \frac{mv^2}{2}}{\frac{mv^2}{2}} = \frac{\frac{9mv^2}{50}}{\frac{mv^2}{2}} = 9\%$$

Ques. $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\left(\frac{l}{g}\right)^{1/2}$

change $$\boxed{\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta l}{l}\right) - \frac{1}{2}\left(\frac{\Delta g}{g}\right)}$$

Error in cal. $$\boxed{\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta l}{l}\right) + \frac{1}{2}\left(\frac{\Delta g}{g}\right)}$$

Ques. T, l change given $\frac{\Delta g}{g} = ?$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$g = \frac{4\pi^2 l}{T^2}$$

$$\boxed{\frac{\Delta g}{g} = \left(\frac{\Delta l}{l}\right) - 2\left(\frac{\Delta T}{T}\right)}$$

error $$\boxed{\frac{\Delta g}{g} = \left(\frac{\Delta l}{l}\right) + 2\left(\frac{\Delta T}{T}\right)}$$

Error in Measurement

① $x = A \pm B$

Δ $x = A + B$ Δ $x = A - B$

Ans. $x \pm \Delta x$

- $x = A \pm B$
- $\boxed{\Delta x = \Delta A + \Delta B}$

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Note:- In addition & substraction absolute error in Result is equal to sum of absolute error in the quantity involved.

(2) $x = AB$ or $\frac{A}{B}$ ($V = IR$, $F = ma$ etc.)

- $x \pm \Delta x \Rightarrow \boxed{x\left(1 \pm \frac{\Delta x}{x}\right)}$

(a) $x = AB$ or $x = \frac{A}{B}$

(b) $$\boxed{\frac{\Delta x}{x} = \frac{\Delta A}{A} + \frac{\Delta B}{B}}$$

Note:- In Product/ Division fractional error in result is equal to sum of fractional error in quantities involved.

(3) $\frac{1}{x} = \frac{1}{A} + \frac{1}{B}$ ($\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$, $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ etc.).

- $x + \Delta x$

- $x^{-1} = A^{-1} + B^{-1}$

diff. w.r.t to variables.

$$\boxed{\frac{\Delta x}{x^2} = \frac{\Delta A}{A^2} + \frac{\Delta B}{B^2}}$$

Ques:- Initial temp. of a body is given by $(40 \pm 0.2)°C$ after some time its temp. is recorded $(60 \pm 0.3)°C$ what is rise in temp within proper error limits.

$T_1 = 40 \pm 0.2$

$T_2 = 60 \pm 0.3$

$$\boxed{\Delta T = T_2 - T_1}$$

$\Delta T = 60 - 40 = 20$

$= T + \Delta T$

$= (20 \pm 0.5)°C$

Ques

$l = (3 \pm 0.03)$ cm

$b = (4 \pm 0.04)$ cm.

Find Area within proper error limits.

$A = l \times B$. $= 3 \times 4 = 12\,cm^2$

$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$.

$= \frac{0.03}{3} + \frac{0.04}{4}$

$= 0.01 + 0.01$

$\frac{\Delta A}{A} = 0.02$

$\Delta A = 0.02 \times 12$

$\Delta A = 0.24\,cm^2$

Ans $= (12 \pm 0.24)\,cm^2$.

Ques. Particle covers ~~15m~~ a dist. of (15 ± 0.5)m in time $t = (3 \pm 0.3)$ sec. cal. spd. within proper error limits.

soln

$d = s \times t$

$s = \frac{d}{t}$ $s = \frac{15}{3} = 5$

$\frac{\Delta s}{s} = \frac{\Delta d}{d} - \frac{\Delta t}{t}$

$\frac{\Delta s}{s} = \frac{0.5}{15} + \frac{0.3}{3}$

$\frac{\Delta s}{s} = \frac{0.1}{3} + \frac{0.3}{3}$

$\frac{\Delta s}{s} = \frac{0.2}{3}$ $\frac{\Delta s}{s} = \frac{2}{3}$ $\frac{0.6}{3}$

~~$\Delta s = 0.66 \times 5$~~ $\Delta s = 0.66$.

$= (5 \pm 0.66)$ m/s.

Ques. $x = \dfrac{a^3 b^2}{c\sqrt{d}}$

% error in a, b, c, d are 1%, 2%, 3%, 4%

$$\cancel{x\%}\ \frac{\Delta x}{x} = 3\left(\frac{\Delta a}{a}\right) + 2\left(\frac{\Delta b}{b}\right) + \frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}$$

$$= 3\times1 + 2\times2 + 3 + \frac{1}{2}(4)$$

$$= 3 + 4 + 3 + 2$$

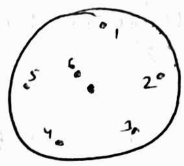
$$= 12\%$$

LEAST COUNT

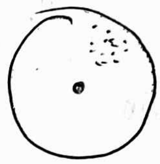
- it is defined for a instrument only.
- it is the smallest value of a physical quantity which can be measured accurately with an instrument.
-

1.5m		
1.50 m	Most accurate	
1.500 m	1.5000	
1.5000 m		

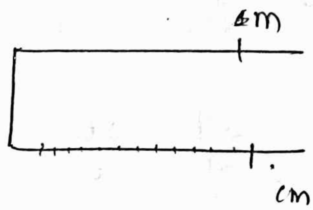
Accuracy



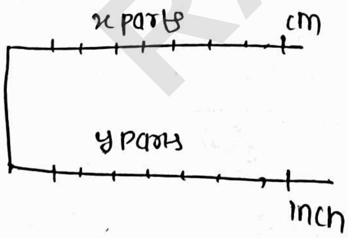
Precision



1s
1.0
1.00 s



1m = 100 (cm)
1m = 100 cm.



length of 1 part ~~of~~ on cm = C
" " " " on inch = I.

$$x \times C = y \times I.$$

$$\boxed{Cm = \frac{y}{x} \times Inch}$$

$$\boxed{Inch = \frac{x}{y} \times cm}$$

Vernier callipers :-

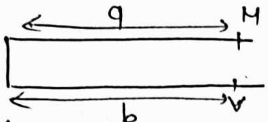
- Two scales
  i) Vernier scale
  ii) Main scale

→ least count (LC)

$$\boxed{LC = M - V}$$

M = Length of 1 part on main scale.
V = " " " " " vernier scale.

•



length of a parts on Main scale = length of b parts on vernier scale

$$\boxed{aM = bV}$$ Test में गलती

* M' given in ques.

$$L.C = M - V$$
$$= M - \frac{aM}{b}$$
$$\boxed{L.C = \left(\frac{b-a}{b}\right)M}$$ (Preferred)

* 'V' is given in ques.

$$L.C = \frac{bV}{a} - V$$
$$\boxed{L.C = \left(\frac{b-a}{a}\right)V}$$

Q. length of 1 part of M.S is 0.1 cm, if 'n' parts of M.S concide with (n+2) parts of V.S then find least count.

$$L.C = \left(\frac{b-a}{b}\right)M \qquad n \times 0.1 = V \times n+2$$

$$nM = (n+2)V$$
$$L.C = M - V$$
$$= M - \frac{nM}{n+2}$$
$$= \frac{Mn+2M-nM}{n+2}$$
$$= \frac{2M}{n+2}$$

$$L.C = 0.1 - \frac{0.1n}{n+2}$$
$$= \frac{0.1(n+2) - 0.1n}{n+2}$$
$$= \frac{0.1n + 0.2 - 0.1n}{n+2} = \frac{0.2}{n+2}.$$

Q. M.s of V.c is divided into several parts such that 1 cm is divided in 10 equal parts if 20 divisions on M.s concide with 25 divisions on V.c scale find L.C.

$20 \times 0.1 = 25 \times V$

$$L.C = M - V$$

$$= 0.1 - \frac{20}{25}$$

$$= \frac{25-20}{25}$$

$$L.C = \frac{0.5}{25} = \frac{0.1}{5} = 0.20 \text{ cm.}$$

$20M = 25V$

~~20x~~

$M \times V$

$M = \frac{25V}{20}$

$M - \frac{20M}{25}$

$\frac{5M}{25}$

$\frac{5}{25} \times 0.1$

Q. ~~screw g~~

Screw guage and spherometer

Two scales.

→ linear scale

→ circular scale

* Pitch :- It is linear dist. covered in 1 rotation

$$\boxed{P = \frac{\text{Total linear dist. covered on linear scale}}{\text{No. of rotations given.}}} = \frac{\text{dist}}{\text{rotation}}$$

* least count :-

$$\boxed{L.C = \frac{\text{Pitch}}{\text{No. of division on circular scale}}} = \frac{P}{}$$

Que. In a screw guage pitch is 1 cm if no. of divisions on circular scale is 100 find L.C.

soln

$$L.C = \frac{1}{100} = 0.01 \text{ cm.}$$

Que. circular scale of a spherometer is divided in 200 equal parts when linear scale is rotated through 20 rotations it moves 2 cm. dist. find L.C

$$\text{Pitch} = \frac{2}{20} = 0.1$$

$$L.C = \frac{0.1}{2000} = \frac{10000}{2000} \times \frac{1}{10000} = \frac{5}{10000} = 0.0005 \text{ cm}$$

* Imp concept.

1. With ↑es in L.C of any instrument the accuracy of measurement ↑es. and error in measurement ↑es.

2. Least count is given for any instrument

   = Minimum possible error.

3. * eq ⇒ $h = 1.25 \times 10^{-2}$ m

   What is permissible error.

   Soln $h = 0.0125$ m

   $\boxed{\Delta h = 0.0001 m}$

   eq. ⇒ $g = 9.80$ m/s²

   $\boxed{\Delta g = 0.01 m/s^2}$