Work, power, Energy

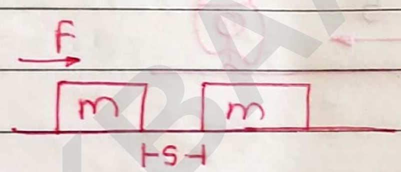
Work :

It a force is applied on a body (point object) hense it's displaced by an amount as disp is s then work done is $\underline{F \times s}$ where s is the disp. caused by these force and its a scalar quantity & it's unit is joule

It's dimension ML^2T^{-2}.

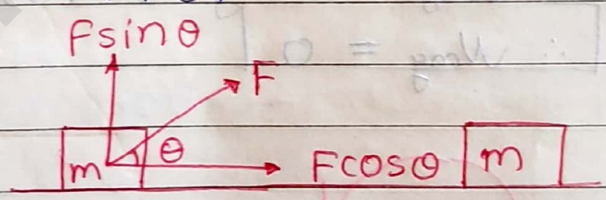
When a body is of mass m placed on table and force is applied unit is displaced by some amount 's' then the value of work is always

Case 1 :



$$W = F \times s.$$

Case 2 :

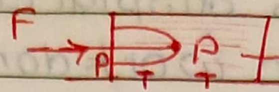


$$w = F\cos\theta \times s$$

$$W = \vec{F}.\vec{S}$$

scalar, Joule & ML^2T^{-2}

Example:

Case  sponge.

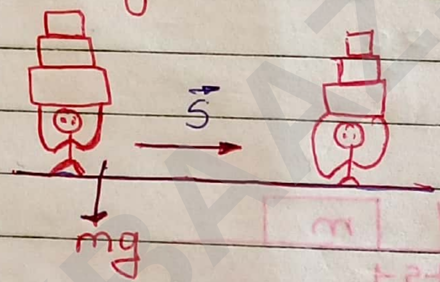
work

(1) 0 (2) $w \neq 0$ ✓ (c) None

work = ~~.~~ force × disp. of point of applica

Cases having zero workdone:

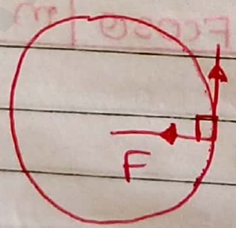
(1)



$W_{mg} = mg \times s \times \cos 90^\circ$

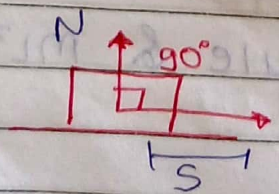
$\boxed{\therefore W_{mg} = 0}$

(2)

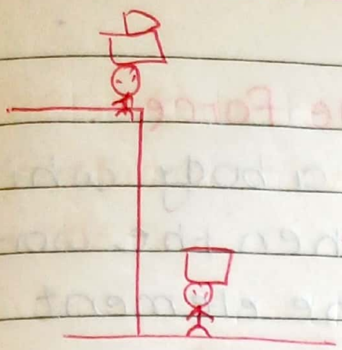


$W_{CPF} = 0$

(3)



$W_N = 0$



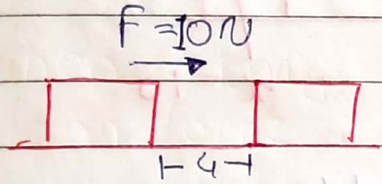
~~w=0~~

g=0

N=0

$W_N = 0$

Q1

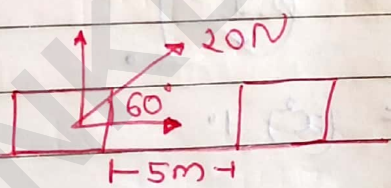


$W = F \times S$

$= 10 \times 4$

$= 40J.$

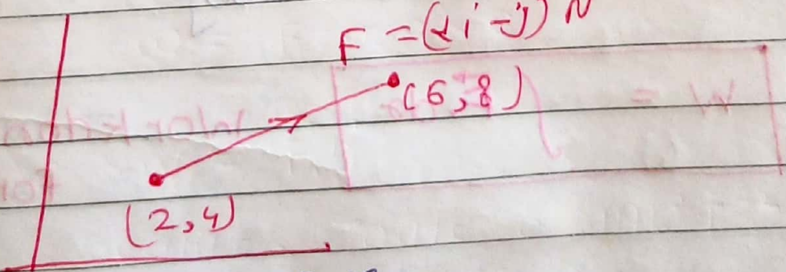
Q2



$W = 20\cos 60^\circ \times 5$

$= 10 \times 5 = 50N.$

Q3



$W = F \cdot r$

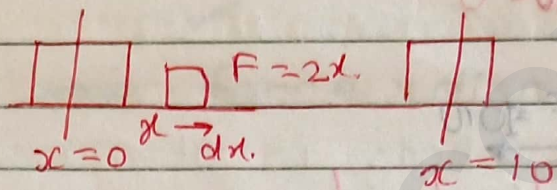
$= (2i - j)(4i + 4j)$

$= 8 - 4$

$= 4$ joule.

Workdone by a barriable force

It a force acts on a body which barries with position then the workdone will be calculated by the element integration method as explain below:



$$\int dw = \int F \cdot dx$$

$$w = \int_0^{10} 2x \cdot dx$$

$$= \left(\frac{\cancel{2}x^2}{\cancel{2}}\right)_0^{10}$$

$$= \left(x^2\right)_0^{10}$$

$$w = 100 J$$

$\therefore$ workdone $= 100 J$

$$\boxed{W = \int \vec{F} \cdot d\vec{r}}$$

Workdone by variable force.

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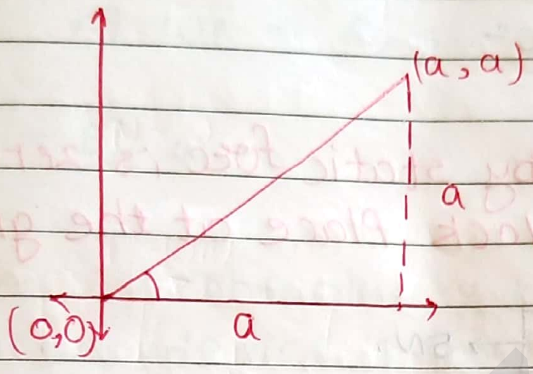
★ When the force is having variable in the expression then workdone will be calculated as under.



$F = y\hat{i} + x\hat{j}$ then find ω

$$\tan \theta = \frac{y}{x}$$

$$\tan \theta = \frac{x}{x} = 1$$

$$\theta = 45^\circ$$

$$\therefore \boxed{y = x}$$

$F = y\hat{i} + x\hat{j}$

$s = (0,0) \rightarrow (a, a)$

$$W = F \cdot r$$

$$\int dw = \int_0^a F \cdot dr$$

$$\int d\omega = \int_0^a (y\hat{i} + x\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$\omega = \int_0^a y\,dx + \int x\,dy$$

$$= \int_0^a y\,dy + \int x\,dx$$

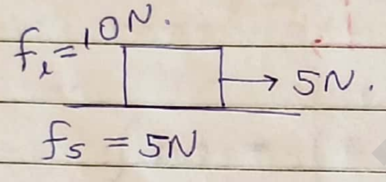
$$w = \frac{y^2}{2} + \frac{x^2}{2}$$

$$= \frac{x^2}{2} + \frac{x^2}{2}$$

$$\therefore w = x^2$$

Workdone by static forc is zero when it acts on a block Place at the ground.

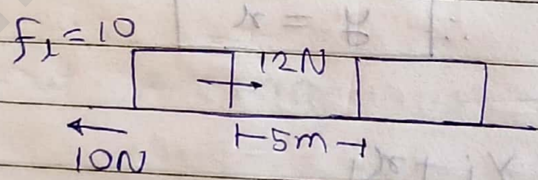
(i)



$\therefore s = 0$

$\boxed{\therefore w = 0}$

Workdone by kinetic friction for a body omoving on ground is always non-zero and negative

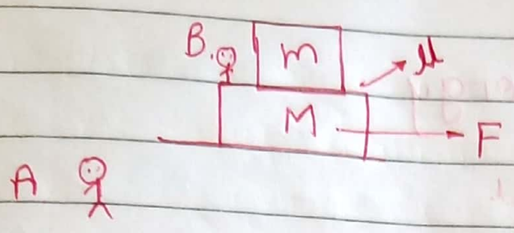


$$w_f = f \times s \cos 180^\circ$$

$$= 10 \times -5$$

$$w_F = -50N$$

$\therefore$ work by friction is -50.



A] $W_F = f \times s$

B] $w = 0$

Energy : The ability to do work of a person, machin, device is known as Energy. in mechanics there are two types of energy.

(1) Kinetic Energy.
(2) Potential Energy.

(1) Kinetic Energy ① The energy of an object / system due to its motion is known as its kinetic energy.

② Its a scalar quantity

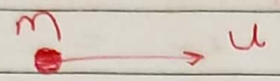
③ Its unit is Joule.

④ If an object of mass m is moving with ~~velo~~ speed B then its kinetic energy will be.

$$\boxed{K.E. = \frac{1}{2} m B^2}$$

If the velocities provided then we can calculate the speed by using the moudles of velocity.

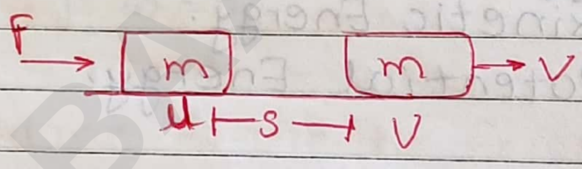
Kinetic Energy



$$K.E = \frac{1}{2} m v^2$$

Work energy theorem

According to these theorem workdon on a body or system is equal to the change in K.E. of body as explained below



① $v^2 = u^2 + 2as$

$$\boxed{\therefore \frac{v^2 - u^2}{2a} = as} \quad \text{—— ①}$$

② $f_1 = ma$ —— ②

③ $work = f \times s$

$= mas$

$= m\left(\frac{v^2 - u^2}{2}\right)$

$work = \frac{1}{2} m v^2 - \frac{1}{2} m u^2$

$\omega = K_f - K_i$

$\omega = \Delta K$

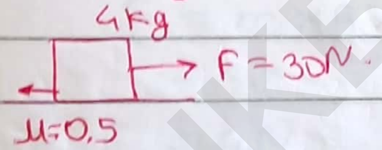
work

Solve the following

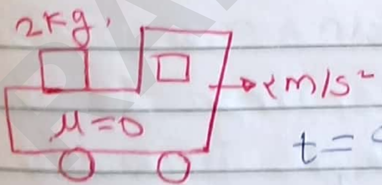
1. If the disp. x of a body depends upon time as then find the work done during time $t=0$ to $t=4$, mass $= 2$

$$x = \frac{t^3}{3}$$

2. If $v = a\sqrt{x}$ of body then find the work done having $x=0$ to $x=d$.

3. 

work. a) F in 1^{st} 2sec.
(b) f in 1^{st} 2sec
(c) K.E. after 2sec

4. 

$t = 4$sec

Ans:

① $x = \frac{t^3}{3}$

$\frac{dx}{dt} = \frac{2t^2}{3} = v = t^2$ $t=0\ v=0$

$t = 4\ v = 16 sec$

$w = \frac{1}{2} m v^2$

$w = \frac{1}{\cancel{2}} \times \cancel{2} \times 16 \times 16$

$w = 256 J$

② $v = a\sqrt{x}$

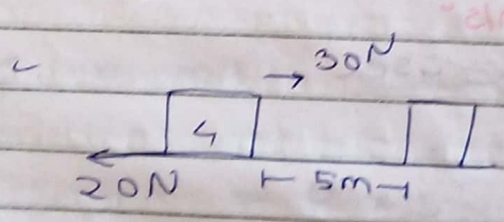
$x = 0 \quad u = 0$

$x = d \quad u = a\sqrt{d}$

$w = \frac{1}{2} m v^2$

$= \frac{1}{2} m a^2 d$

③



① $a = \frac{10}{4} = 2.5 m/s^2$

$s = \frac{1}{2} a t^2$

$= \frac{1}{2} 2.5 \times (2)^2 = \frac{1}{\cancel{2}} 2.4 \times \cancel{4}^2$

$= 4.8 \approx 5$

(2) $W_F = F \times S$

$= 30 \times 5$

$= 150\,J$

(3) $W_f = 20\cos 180° \times 5$

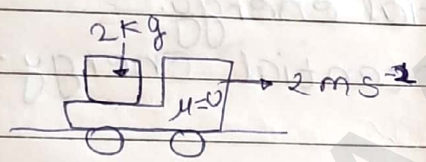
$= -20 \times 5$

$= -100\,J$

$K.E. = 150 - 100$

$= 50\,J$

(4)



$F = m \times a$

$= 2 \times 2$

$F = 4\,N$

$a = 2\,m/s^2$

$S = \frac{1}{2} a t^2$

$= \frac{1}{\cancel{2}} \times \cancel{2} \times (4)^2$

$S = 16\,m$

$w = \overset{S}{16} \times \overset{F}{4}$

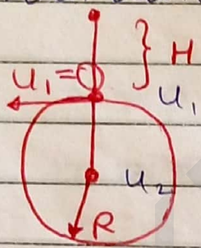
$= 64$

Potential Energy, conservative & non-conservative forces, types of equilibrium & force & potential energy releation and Energy stored in a spring.

1. **Potential Energy:** The energy in a body/system due to its specific position is known as its potential energy.

• In the syllabus of neet we discuss 4 types of potential energy.

(1) Gravitational potential Energy:

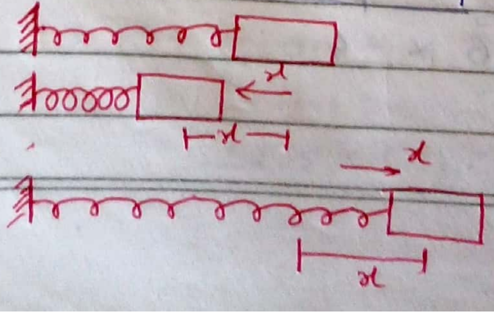


$u_1 = mgh$

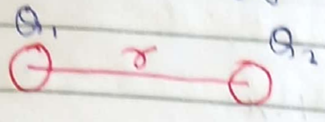
$$u_2 = \frac{mgh}{1 + h/R} = \frac{Rmgh}{1+h}$$

(2) Potential Energy of spring: If a spring of force constant k is provided which is compress or elongated by an amount x then its potential energy is given by the expression

$$\boxed{\therefore u = \frac{1}{2} kx^2}$$



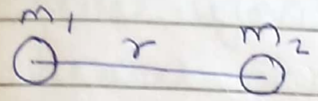
③ pontential energy of two chargy stystem



$$u = \frac{kQ_1Q_2}{r}$$

$(k = 9 \times 10^9)$

④ P.E. of two masses system.



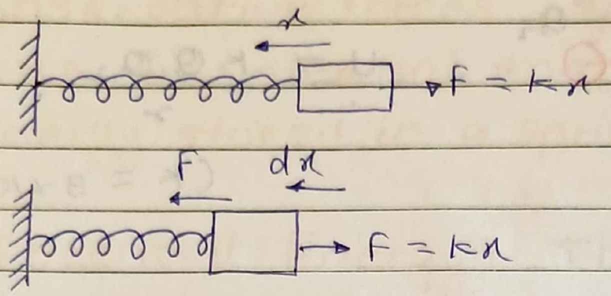
$$u = \frac{-Gm_1m_2}{r}$$

⑤ P.E. of particle or system depends upon the frame of refrence but change in potential energy is independent on frame of reference.

→ When the work is done on a system it will store in the system in the form of potential energy.

→ For example if a spring of force constant k is given & we want to compress it by small amount dx then we have to do the work agains the spring force at these force is store in the system (spring) in form of potential energy.

U for spring force.



$$\int dw = \int F \cdot dx.$$

$$w = \int kx \cdot dx$$

$$w_{spring} = \frac{1}{2} kx^2$$

$$W_{ext.\ ag} = U_{spring}$$

* We know that when a person or a system will do the work it will loose its energy so for the person we can write:

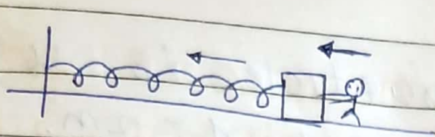
$$dw = -du.$$

For ext. agent / body.

$$\boxed{dw = -du.} \quad \text{—— ①}$$

But for external agent and body

$$\boxed{dw = du}$$

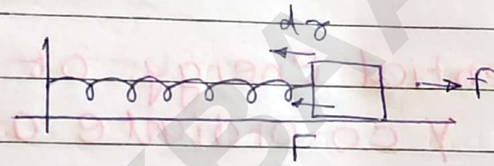


$du = -du$

$\longrightarrow$ $\boxed{dw = du}$

Force & P.E. Releation

We know that The workdone by an external agent will be :- $dw = F.dr$
We know that workdone is equal to to loss of enegy so by using this concept we can derive the relation between force & P.E.



$$dw = F.dr \quad \text{—} ①$$

$$dw = -du \quad \text{—} ②$$

$$-du = F.dr \quad \text{—} ③$$

$$\boxed{\therefore F = \frac{-du}{dr}} \quad \text{—} ④$$

→ from the above eq^n we can conclude that the negative of rate of change of P.E. is equal to force on it.

If P.E. of particle is $u = 2x^3$ then find force on it at $x = 2m$.

$$F = -\frac{du}{dx}$$

$$= -\frac{d(2x^3)}{dx}$$

$$= -6x^2$$

$$= -6(2)^2$$

$$F = -24N$$

If the potential Energy of particle depends upon Y co-ordiante as $u = 3y^2$ then find the force at $y = 4m$

$$F = -\frac{du}{dy}$$

$$= -\frac{d(3y^2)}{dy}$$

$$= -6y$$

$$= -6(4)$$

$$F = -24N$$

If the P.E. is varies in all direction then we canculate the force as under.

$$F = -\ grad.\ u \quad —①$$

$$F = -\vec{\nabla} u \quad —②$$

$$F = -\left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right)$$

$$F = -\left(\frac{du}{dx}\hat{i} + \frac{du}{dy}\hat{j} + \frac{du}{dz}\hat{k}\right)$$

If $u = x^2+y^2+z^2$ then find it's P.E. at $P(1,1,1)$?

$$F = -\left[\frac{d(x^2+y^2+z^2)}{dx}\hat{i} + \frac{d(x^2+y^2+z^2)}{dy}\hat{j} + \frac{d(x^2+y^2+z^2)}{dz}\hat{k}\right]$$

$$= -\left[2x\hat{i} + 2y\hat{j} + 2z\hat{j}\right]$$

$$= -2(x\hat{i} + y\hat{j} + z\hat{j})$$

$$= -2(\cancel{x}\,i + j + k)$$

$$= -2(\sqrt{3})$$

$$|F| = -2\sqrt{3}\ N$$

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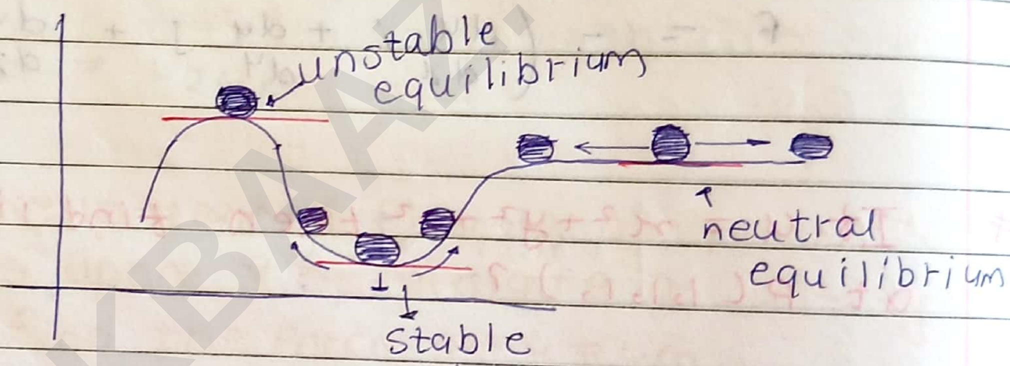
Equilibrium & ~~involveds~~ it types:

If a body is in equilibrium then net force will be zero

→ If a P.E. and $\vec{r}$ (position) graph is provided then there are three types of equilibrium which can be describe by using these graph as explained below:



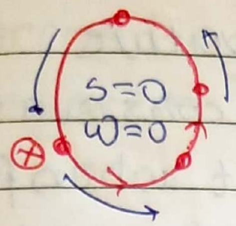
slope = 0 , $-\frac{du}{dx} = 0$

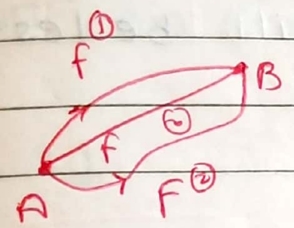
$F = \frac{-du}{dr}$

Conservative & non-conservative forces:

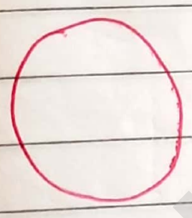
(1) conservative forces : If a work done in a closed path by a force is zero then force is called conservative force means under these forces the amount of workdone depends upon disp- & under these forces the change

K.E. in a closed path is zero.





$W_1 = W_2 = W_3$



$W = \Delta K$

In a closed path

$W = 0$

so $\Delta K = 0$

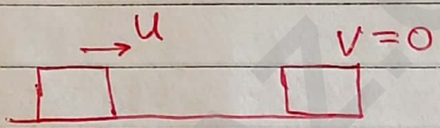
$K_f - K_i = 0$

$K_f = K_i$

Non conservative forces : If a workdone in a closed path is non-zero then the force is called non conservative

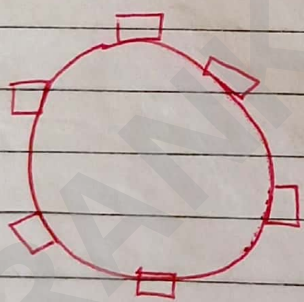
e.g. Friction is a non-conservative force because the energy is lost when friction acts on a body.

→ Because the energy is lost so the final kinetic energy will be lesser than initial kinetic energy.



$f = \mu mg$

$$a = \frac{\mu mg}{m} = \mu g$$



$$W_{closed} = f \times 2\pi r$$

$$K_2 = K_1 - K_f$$

$$K_2 = K_1 - (\mu mg) \times 2\pi r$$

$$K_2 < K_1$$

Principle of conservation of energy. (PCOE)

According to these law the total energy of a system remains conserve / constant. some how we can convert it from one form to anothee form.

e.g. The K.E. is convert in P.E. and P.E. can convert in to the K.E.

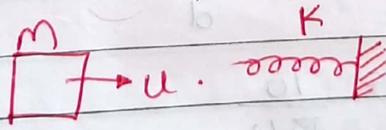
foe e.g.

$u = mgh$

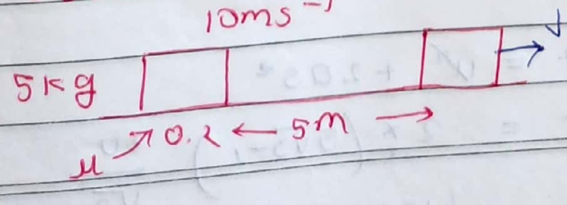
$(T.E.)_A = mgh$

$$(T.E.)_B = \frac{1}{2} m \left(\sqrt{2gh}\right)^2$$
$$= \frac{1}{\cancel{2}} m \cancel{2} gh$$
$$= mgh.$$

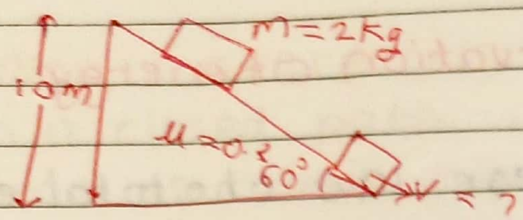
If a block of mass m moving with a velocity u as shown in fig. then find the maximum compression in spring.



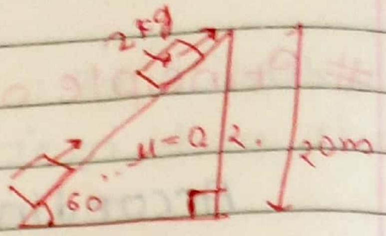
For a given situation find velocity of block after travelling 5m if $\mu = 0.2$



Q3

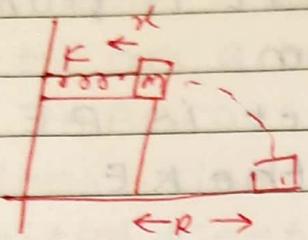


Q6

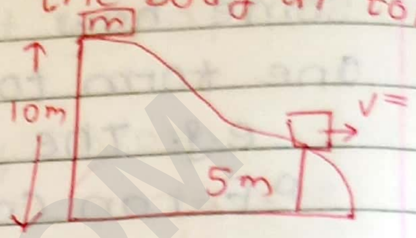


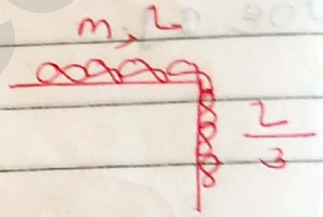
woeF to find the body at top.

Q4



Q5





Q3 →

$f \leftarrow \square \rightarrow mg\sin 60^\circ$

$\mu mg\cos 60^\circ$

$f_1 = 0.2 \times \cancel{2} \times 10 \times \frac{1}{\cancel{2}}$

$f_1 = 2$

$F = \frac{\cancel{2} \times 10 \times \sqrt{3}}{\cancel{2}} = 10\sqrt{3}$

$\text{accn.} = \frac{\overset{5}{\cancel{10}}\sqrt{3} - \cancel{2}}{\cancel{2}}$

$= 5\sqrt{3} - 1$

~~tan~~ $\sin 60^\circ = \frac{10}{d}$

$d = \frac{10}{\sqrt{3}/2}$

$d = \frac{20}{\sqrt{3}}$

$v^2 = u^2 + 2as$

$v^2 = 2 \times (5\sqrt{3} - 1)\frac{20}{\sqrt{3}}$

$= (10\sqrt{3} - 2)\frac{20}{\sqrt{3}} \qquad \frac{10\cancel{\sqrt{3}} \times 10}{\cancel{\sqrt{3}}} - \frac{20}{\sqrt{3}}$

$$\frac{10\sqrt{3} \times 10 - 20}{\sqrt{3}} \qquad 100 - 12 = 88$$

$$\frac{100\sqrt{3} - 20}{\sqrt{3}}$$

$$\frac{173 - 20}{\sqrt{3}}$$

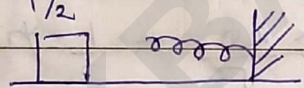
$$= \frac{\cancel{17}\ 153}{\sqrt{3}}$$

$$= \frac{153}{1.73}$$

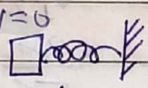
= 809 m/s

Q1

$\frac{1}{2} mu^2$



$V = 0$



$\frac{1}{2} kx^2$

$$\cancel{x = \frac{2}{k}} \qquad \frac{1}{2} mu^2 = \frac{1}{2} kx^2$$

$$x^2 = \frac{mu^2}{k}$$

$$x = \sqrt{\frac{mu^2}{k}}$$

Q.② $\frac{1}{2}mu^2 = W_{ag.f} + \frac{1}{2}mv^2$

$\frac{1}{\cancel{2}}\cancel{m}u^2 = 2\mu \cancel{m}g \times S + \frac{1}{\cancel{2}}\cancel{m}v^2$

$(10)^2 = 2(0.2) \times 10 \times 5 + v^2$

$v^2 = \frac{\cancel{100}}{\cancel{10 \times 0.4 \times 5}} \; 100 - 20$

$v^2 = 80$

$v = \sqrt{80}$

Q.③ $mgh = W_{ag.f} + \frac{1}{2}mv^2$

$\cancel{m}gh = (\mu \cancel{m}g\cos\theta) \times \frac{20}{\sqrt{3}} + \frac{1}{2}\cancel{m}v^2$

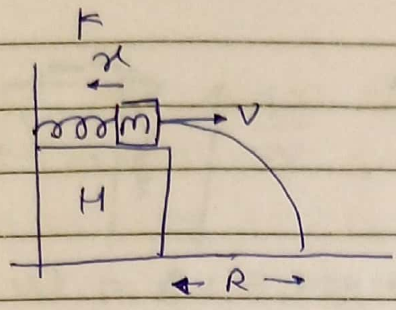
$100 = \cancel{0.2} \times 1\cancel{0} \times \frac{1}{\cancel{2}} \times \frac{\cancel{20}}{\cancel{\sqrt{3}}} + \frac{v^2}{2}$

$88 \times 2 = v^2$

$v^2 = 176$

$v = \sqrt{176}$

$v = 13.3$



$$\frac{1}{2} kx^2 = \frac{1}{2} mv^2$$

$$v^2 = \frac{kx^2}{m}$$

$$v = x\sqrt{\frac{k}{m}}$$

$$R = vt$$

$$= x\sqrt{\frac{k}{m}} \times \sqrt{\frac{2h}{g}}$$

$$= x\sqrt{\frac{2kh}{mg}}$$

Power : The rate of workdone by a machine, device or a person is known as its power.

① The unit of power is watts or joule/sec.

② Its a scalar quantity and its dimension is ML^2T^{-3}

$$P = \frac{W}{t} = \frac{F.S}{t} = \vec{F}.\vec{V}$$

The person, device, machine does more work in less time is called more powerful.

(1) If $\omega = 2t^2$ —① power at 3 sec

$$P = \frac{d\omega}{dt} = 4t = 4(3)$$
$$= 12.$$

(2) If $P = 2t$ then find the the power from $t=0$ to $t=4$ sec.

$$\langle P \rangle = \frac{\int P.dt}{\int dt}$$

$$= \frac{\int_0^4 2t.\,dt}{\int_0^4 dt}$$

$$= \frac{(t^2)_0^4}{(t)_0^4}$$

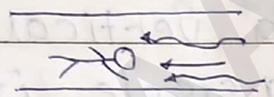
$$= \frac{16}{4} = 4 \text{ watt.}$$

If a swimmer is swimming is in opposite direction of flower and the resistance offer by the water is $\frac{1}{5}$ times of total weight then how much power he has to deliver with constant velocity. If his mass is 50 kg.

If a constant power is applied on an object so its start moving then prove that the distance covered by it in time t will be $s \propto t^{3/2}$

MTR

Ans ① :



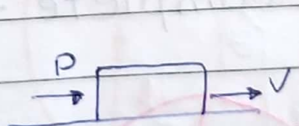
$$F = \frac{mg}{5} =$$

$$P = F.V. = \frac{mg}{5} \times V$$

$$= \frac{50 \times 10}{\cancel{5}} \times \cancel{5}$$

$$= 500 \text{ watt}.$$

②



$$P = \frac{W}{t} = \frac{1}{2}\frac{mV^2}{t}$$

$$V^2 = \frac{2Pt}{m}, \quad V = \sqrt{\frac{2Pt}{m}}$$

constant

$$S = Vt$$

$$S = \sqrt{\frac{2Pt}{m}} \times t = \sqrt{\frac{2P}{m}} \times t^{1/2+1} \times t^{1}$$

$$\sqrt{\frac{2P}{m}} \; t^{3/2}$$

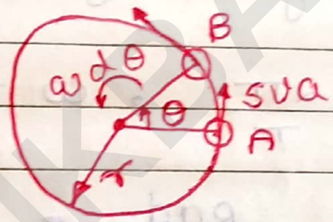
Circular Motion.

→ If a particle moves such that its distance from a fixed point always remain constant then such a motion called circular motion.

→ These fixed point is called centre and these distance is called radius.

→ In a circular motion the particle moves on the circumference along the tangent so it can be treated as linear motion.

→ Where as at the centre the motion is angular motion and these two can be releated with each other as explained below:



$$\theta = \frac{s}{r}$$

$$s = r\theta$$

$$\frac{ds}{dt} = \frac{dr}{dt}\frac{d\theta}{dt}$$

$$v = r\omega$$

$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

$$a = r\alpha$$

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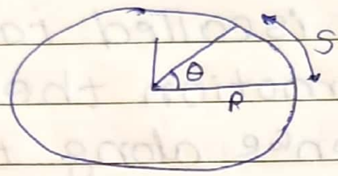
★ Impoetant defination.

1 Angular disp. : Angle subtented at the center is called angular disp. it's represented by θ



$$\theta = \frac{S}{R}$$

2. Angular velocity: Angular disp. of a body in unit time is known as angular velocity it's represented by the symbol 'ω' and its unit is rad./sec.

$$\omega = \frac{2\pi}{T}, \quad \omega = 2\pi N$$

$$\frac{rev.}{sec} = N \qquad \frac{Rad.}{sec} = \omega$$

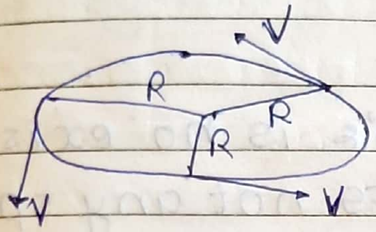
3. Angular accn.: The rate of change of angular velocity is known as angular accn. it's represented by the symbol α. and formular for these.

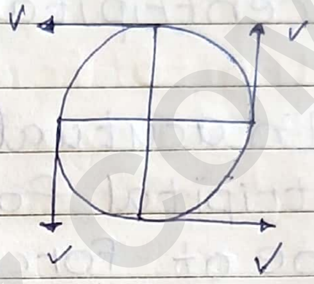
$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

Unit: rad./sec²

$$\alpha = \frac{d\omega}{dt}$$

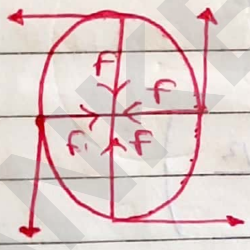
Uniform circular motion : In these type of motion the speed of particle remain constant but velocity is change. at any instant the velocity is always along the tangent so speed & K.E remain constant but velocity change.

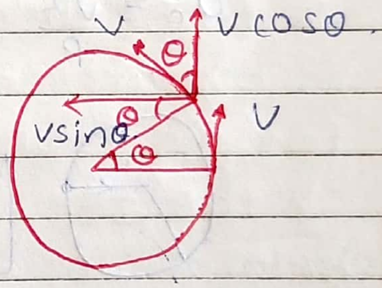




K.E = const.
$= \frac{1}{2} m v^2$.
speed.

In U.C.M. particle will bend continously because a force will act on a body towards the centre which is called centripital force.





$$a_c = \frac{\Delta v}{t} =$$

$$= \frac{v\sin\theta + 0}{t}$$

$$= \frac{v\,\theta}{t}$$

$$a_c = v\omega$$

$$= v \times \frac{v}{R}$$

$$\boxed{\therefore\ a_c = \frac{v^2}{R}}$$

$$a_t = \frac{\Delta v}{t}$$

$$= \frac{v - v\cos\theta}{t}$$

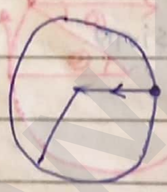
$$= \frac{v - v}{t}$$

$$\boxed{a_t = 0}$$

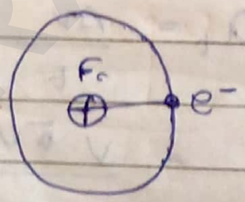
From the above expression we can conclude that for a particle in state of U.C.M. & accn. is their always towards the centre so a force must act towards centre on the particle, these force is known as centripital force.

★ In acutuallay their is no existance of centripital force or its not any special type of force the only thing is the force which acts on the body will have a fixed magnidue which is called $\frac{mv^2}{R}$.

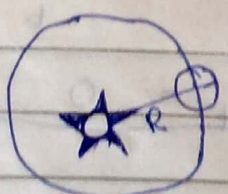
$$a_c = \frac{v^2}{R}$$



$$\boxed{F = \frac{mv^2}{R}}$$



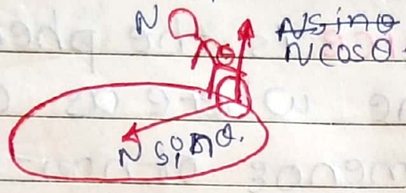
$F_e = mv^2/R$
calombian force



$$F_g = \frac{mv^2}{R}$$

*Bending and Banking of cyclist and tracts.

When a cyclist takes a turn then there is a requirement of centripital force which is provided by normal reaction b/w grown & the cyclist as explained below:



$$N\sin\theta = \frac{mv^2}{R}$$

$$N\cos\theta = mg$$

$$\boxed{\therefore \tan\theta = \frac{v^2}{Rg}}$$

* Centripital & centrifugal.

In inertial frame the circullar motion is discribed as " the requise centripital force will be given by the existing force.".

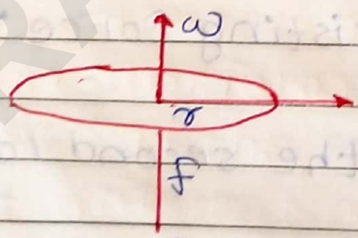
→ These is to satisfy the second law of newton

* Bending of track: To provide C.P.M. to heavy vehicle. one end of the road will be raised by some height which is known as banking of the track.

Centripital force is the phenomenon of the inertial frame where as Centrifugal force is the phenomenon of non-inertial force.

Then cannot act a body together

Whenever the equilibrium of an object is to be discussed then centrifugal force is balanced by the force present there i.e. for ex. columbian force / gravitional force, or the tension in the string as explained below.



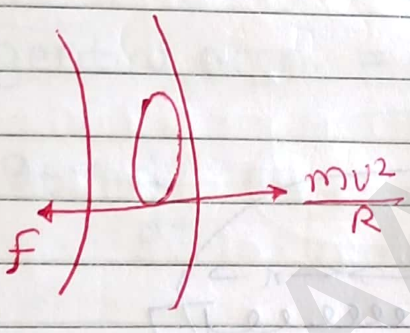
$f = m\omega^2 r$ =

$$\cancel{m}\omega^2 r = \mu \cancel{m} g$$

$$\omega = \sqrt{\frac{\mu g}{r}}$$

Bending of cyclist in case of friction and banking of car in case of friction and corresponding min. & max. velocity.

If a cyclist wants to take a turn without bending on a road then his max. velocity will be $v = \sqrt{\mu r g}$. as explain

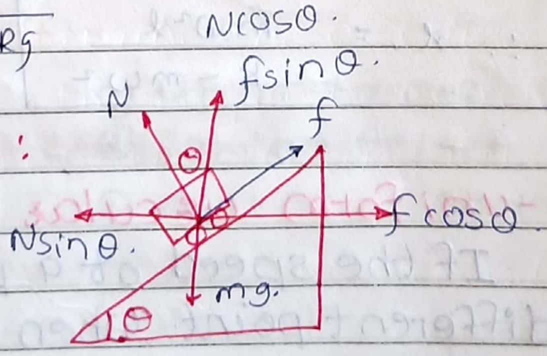


$$f = \frac{mv^2}{R}$$

$$\mu \cancel{m} g = \frac{\cancel{m} v^2}{R}$$

$$v = \sqrt{\mu R g}$$

min. velocity :



$$\cancel{N} \sin\theta - \underset{\mu \cancel{N}}{\cancel{f}} \cos\theta = \cancel{m} v^2/R$$

$$\cancel{N} \cos\theta + \underset{\mu \cancel{N}}{\cancel{f}} \sin\theta = \cancel{m} g$$

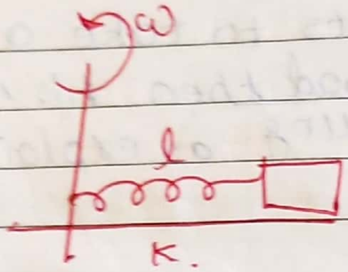
$$\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta} = \frac{v^2}{Rg}$$

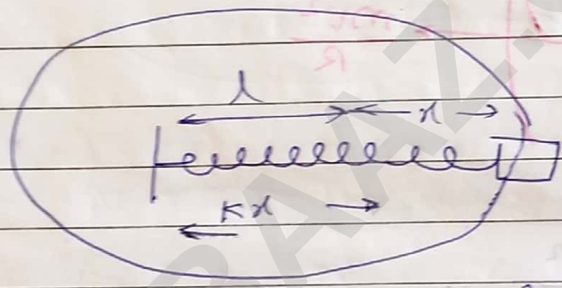
dividing by cos θ.

$$\frac{\tan\theta - \mu}{1 + \mu\tan\theta} = \frac{v^2}{Rg}$$

$$v = \sqrt{\frac{\tan\theta - \mu}{1 + \mu\tan\theta} Rg}$$

The max. aloud velocity on circular turn will be $\sqrt{\frac{Rg(\tan\theta+\mu)}{1-\mu\tan\theta}}$





$$kx = m\omega^2(l+x)$$

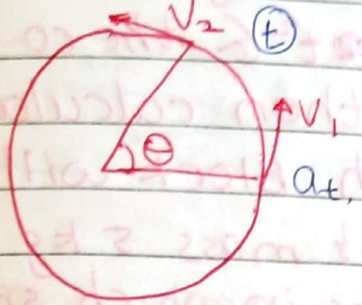
$$kx = m\omega^2 l + m\omega^2 x$$

$$\boxed{\therefore x = \frac{m\omega^2 l}{k - m\omega^2}}$$

Non-uniform circular motion

If the speed of a particle changes at different point when it's revolving in a circle then such a motion is called non-uniform circular motion.

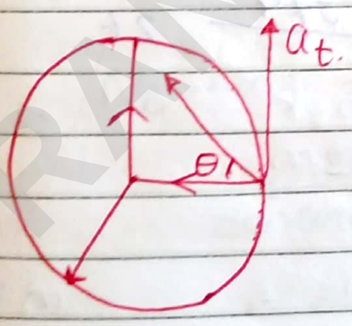
In these motion an additional accn. is produced along the tangent which is responsible to speed up or speed down the particle is known as tangential accn. as explain below.



Tangential accn. $= a_t = \cancel{t_2 - t_1} = \frac{V_2 - V_1}{t} = \frac{dv}{dt}$

$$\boxed{\therefore a_t = \frac{dv}{dt}}$$

tangential accn. : It's rate of change of speed not velocity but it's not in actual a_t.



$v = t^2$

① ~~θ~~ at $t = 1$ sec

② a_c at $t = 1$ sec.

③ $a_{vct} = ?$ at $t = 1$ sec.

$a_N = \sqrt{a_c^2 + (a_t)^2}$

$= \sqrt{4+1} = \sqrt{5}$

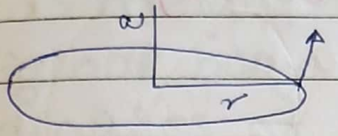
$a_t = \frac{dv}{dt} = 2t.$

$a_t = 2 ms^{-1}$

$\tan\theta = \frac{a_t}{a_c}$

$= 2$

cross product teleation v, r & ω & a, r & α

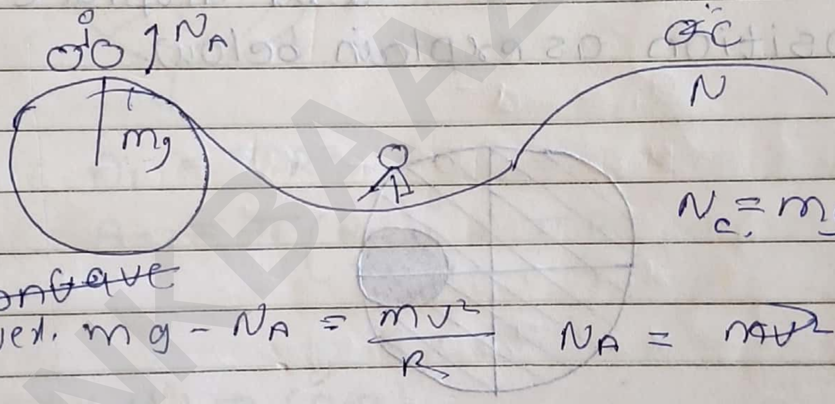


$V = \omega \times R$

$A = \alpha \times R$

convex, concave & plane ~~ring~~ bridge.

If a



$N_C = mg.$

~~concave~~

convex. $mg - N_A = \frac{mv^2}{R}$ $N_A =$ ~~mv²~~

$N_B - mg = mv^2R.$

$N_B = mg + mv^2R.$

concave.

$N_B > N_C > N_A$

Verticle Circular Motion

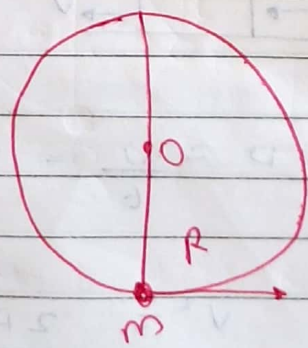
Vertical circulae motion

→ When a body revelove in a vertical motion then its motion is called vertical circulae motion

→ When a body revelove in a vetical circle a minimum velocity is required at the lowest point as well as heightest point which is known as critical velocity.

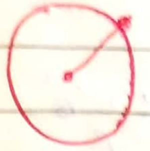
If a we tied stone with a string & revelouing in a vertical circle the it will complete a circle it tension is non-zero at the heighest point. even the body with complete the circle it tenstion just becomes zero at the heighest point.

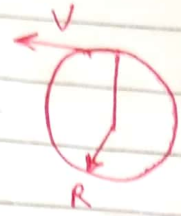
The condition of V.C.M tension in the string must be non-zero upto the highest point then body will complete the circle.



MTR

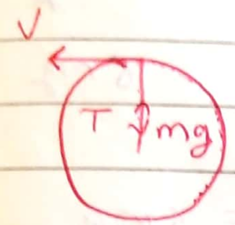
velocity → P CoE.
tension → F.em





T = centripital force.

Inclined case (when body just complete the circle)



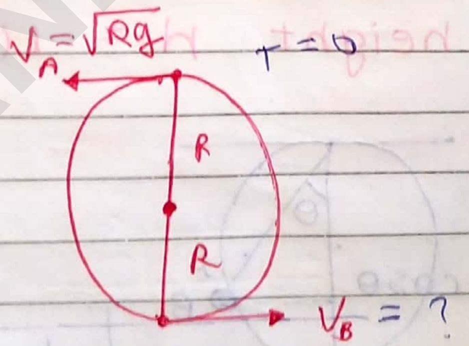
$$F_{net} = \frac{mv^2}{R}$$

$$T + mg = \frac{mv^2}{R} \quad -(1)$$

$$0 + \cancel{m}g = \frac{\cancel{m}v^2}{R}$$

$$v = \sqrt{Rg}$$

To calculate the velocity at the lowest point we will use PCOE.



V = 0 (Total Energy)

$$(TE)_A = (TE)_B \quad -(1)$$

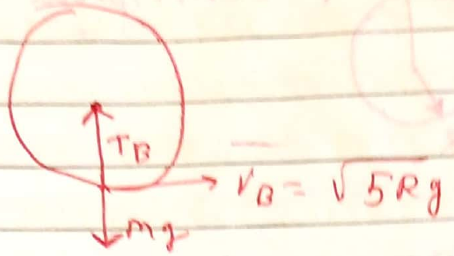
$$U_a + K_a = 0 + \frac{1}{2}mv_B^2$$

$$\cancel{m}g(2R) + \frac{1}{2}\cancel{m}(Rg) = \frac{1}{2}\cancel{m}v_B^2$$

$$\frac{5\cancel{m}gR}{\cancel{2}} = \frac{1}{\cancel{2}}\cancel{m}v_B^2$$

$$5gR = v_B^2$$

$$v = \sqrt{5gR}$$



$$T_B - mg = \frac{mv_B^2}{R}$$

$$T_B = \frac{mv_B^2}{R} + mg$$

$$T_B = mg + \frac{m}{\cancel{R}} \times 5\cancel{R}g$$

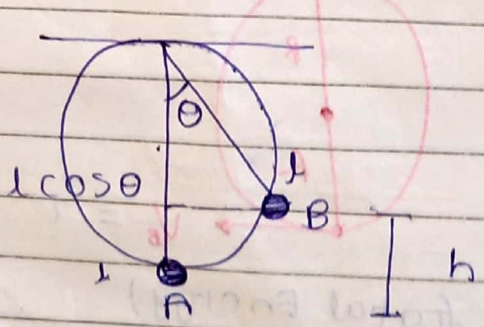
$$T_B = 6mg$$

$$T_B - T_A = 6mg.$$

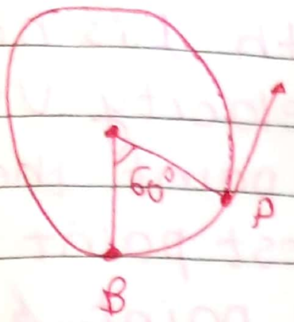
Always.

Velocity and tension at angle θ.

★ If a pendulum is displaced by an angle θ then it will lift up from the ground to height $h = l(1-\cos\theta)$



$$h = l - l\cos\theta.$$



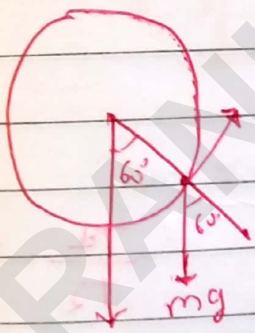
$$(T E)_B = (T.E)_P$$

$$0 + \frac{1}{2} m 5Rg = mg \frac{R}{2} + \frac{1}{2} m V_B^2$$

$$2 \cancel{m} g R = \frac{1}{2} \cancel{m} V_P^2$$

$$V_P^2 = 4Rg$$

$$V = \sqrt{4Rg}.$$



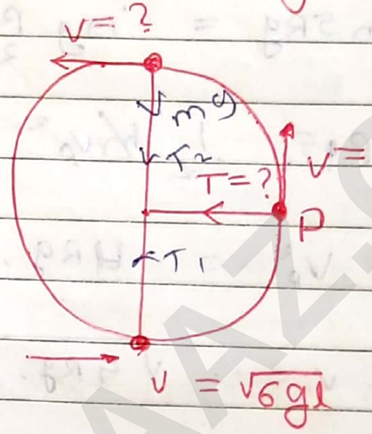
$$T_P - mg\cos 60^\circ = \frac{m 4 \cancel{R} g}{\cancel{R}}$$

$$T_P = \frac{4mRg + mg}{2R}$$

$$4mg + \frac{1}{2} mg.$$

$$T_P = \frac{9}{2} mg.$$

If a pendulum of length l is hanging as shown in fig and a velocity $v = \sqrt{6gl}$ is given at the lowest point then find the velocity at the heighest point and tension at the heighest point & also find tension and velocity at point P.



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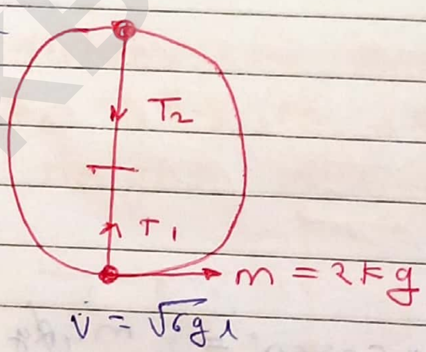
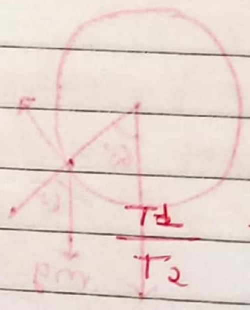
$T_1 - mg = \frac{m}{\cancel{l}} 6g\cancel{l}$

$T_1 = 7mg$

$T_1 - T_2 = 6mg$





$\frac{T_1}{T_2} = ?$

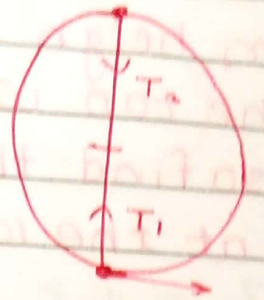
① →

$$\frac{1}{\cancel{2}} \times \cancel{m} \times \cancel{6}^{3} gl + 0 = \cancel{2}\cancel{m}gl + \frac{1}{2}\cancel{m}v^2$$

$$3gl = 2gl + \frac{1}{2}v^2$$

$$gl = \frac{1}{2}\cancel{m}v^2$$

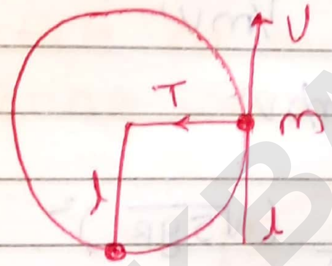
$$v = \sqrt{2gl}$$



$$T_2 + mg = \frac{mv^2}{l} = \frac{m2g\cancel{l}}{\cancel{l}}$$

$$T_2 = 2mg - mg$$

$$T_2 = mg$$



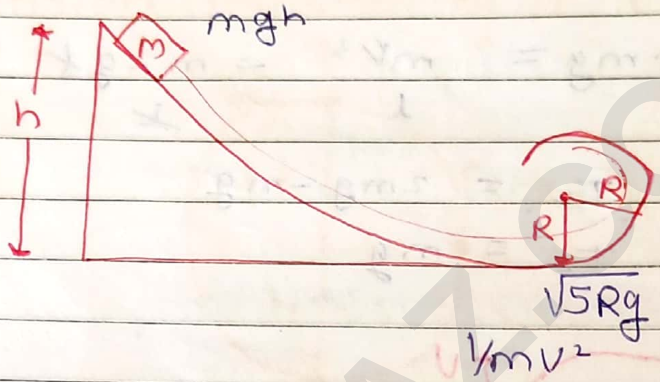
$$2mgl = \frac{1}{2}mv^2$$

$$v = \sqrt{4gl}.$$

$$\frac{1}{2}m \times 6gl + 0 = mgl + \frac{1}{2}mv^2$$

$$T_3 = \frac{mv^2}{l} \quad T_3 = \frac{m4gl}{l} - 4mg.$$

Find the minimum heigh for which body release from the top will complete the circle. and also find the value of normal reaction at the lowest point.



$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$$

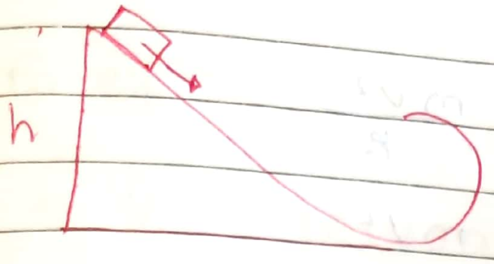
$$gh = \frac{1}{2}(\sqrt{5gR})^2$$

$$\cancel{g}h = \frac{1}{2}5R\cancel{g}$$

$$\underline{h = 2.5R.}$$

$$h = \frac{5}{2}\frac{D}{2}$$

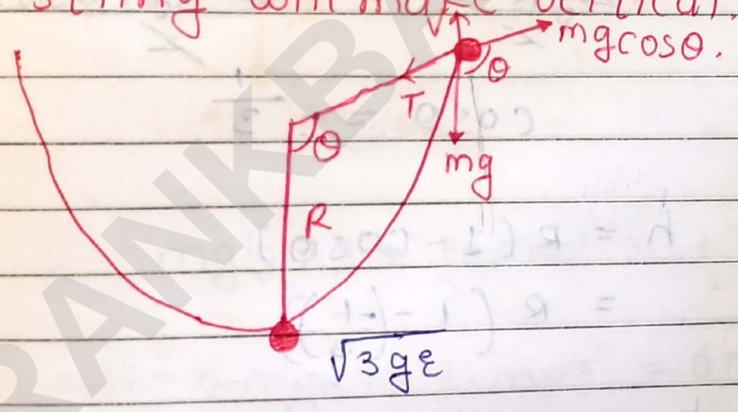
$$h = \frac{5}{4}D$$



R = 6m

$$h = \frac{5}{2}R = \frac{5}{\cancel{2}} \times \cancel{6}\,3 = 15m$$

It a body is hanging by a string as shown if a velocity $v = \sqrt{3Rg}$ It given at the lowest point then find at what height the string become looses & what will be the velcity at a paeticle at that point How much angle the string will make vertical.



$$mg \times 0 + \frac{1}{2}m(\cancel{\ }3Rg) = \frac{1}{2}mV^2 + mgR(1-\cos$$

$$\frac{3mRg}{2} = \frac{1}{2}mV^2 + mgR - mgR\cos\theta.$$

$$\frac{mgR}{2} = \frac{1}{2}mV^2 - mgR\cos\theta.$$

$$\frac{mgR}{2} + mgR\cos\theta = \frac{1}{2}mV^2 \quad \text{—①}$$

$$T - mg\cos\theta = \frac{mv^2}{R} \quad \text{— ①}$$

$T = 0.$

$$-mg\cos\theta = \frac{mv^2}{R}$$

$$mv^2 = -mgR\cos\theta. \quad \text{— ③}$$

on using eq^n ① & ③

$$\frac{mgR}{2} + mgR\cos\theta = \frac{1}{2}(-mgR\cos\theta)$$

$$\frac{mgR}{2} + mgR\cos\theta = -\frac{mgR\cos\theta}{2}$$

$$\frac{\cancel{mgR}}{\cancel{2}} = -\frac{3\cancel{mRg}\cos\theta}{\cancel{2}}$$

$$\cos\theta = -\frac{1}{3}$$

$$h = R(1 - \cos\theta)$$

$$= R\left(1 - \left(-\frac{1}{3}\right)\right)$$

$$h = R\left(1 + \frac{1}{3}\right)$$

$$h = R\left(\frac{4}{3}\right)$$

$$\therefore h = \frac{4R}{3}$$

$2R = \frac{1}{2} g t^2$

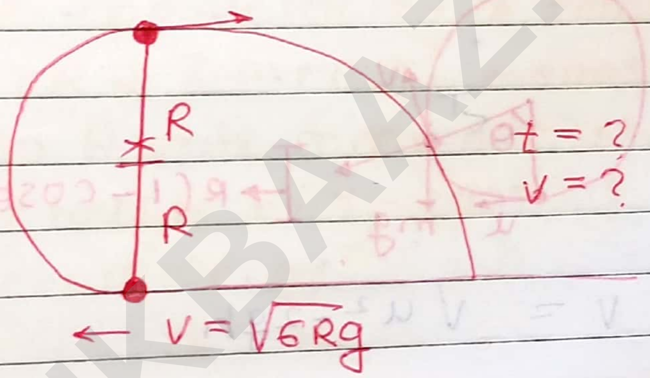
$\frac{4R}{g} = t^2 \qquad t = \sqrt{\frac{4R}{g}}$

for eqn ②.

$$v^2 = gR\cos\theta \quad \text{---} ②$$

$$v^2 = \frac{gR}{3}$$

$$v = \sqrt{\frac{gR}{3}}$$



$$mg(2R) + \frac{1}{2} mv^2 = \frac{1}{2} m\,6Rg$$

$$2mgR + \frac{1}{2} mv^2 = \frac{6mRg}{2}$$

$$\frac{1}{2} \cancel{m} v^2 = \frac{2\cancel{m}Rg}{\cancel{2}}$$

$$v = \sqrt{2Rg}$$

Alternative.

$$v^2 = u^2 - 2gh$$

$$= 6Rg - 2g \times 2R$$

$$v^2 = 2Rg$$

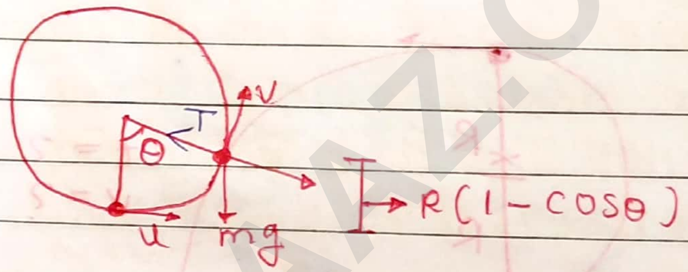
$$v = \sqrt{2Rg}$$

$$t = \sqrt{\frac{2H}{g}}$$

$$= \sqrt{\frac{4R}{g}}$$

$$\text{Range} = v \times t = \sqrt{2Rg} \times \sqrt{\frac{4R}{g}}$$

$$= 2\sqrt{2}\,R$$



$$v = \sqrt{u^2 - 2gh}$$

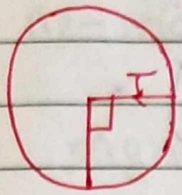
$$T = mg\cos\theta + \frac{mv^2}{R}$$

$$T = mg\cos\theta + \frac{m(u^2 - 2gh)}{R}$$

$$= mg\cos\theta + \frac{mu^2}{R} - 2mg(1 - \cos\theta)$$

$$T = 3mg\cos\theta - 2mg + \frac{mu^2}{R}$$

$$T = mg(3\cos\theta - 2) + \frac{mu^2}{R}$$

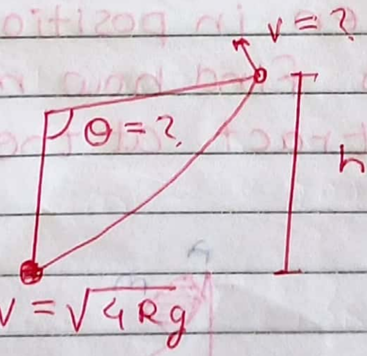


$$T = -2mg + 5mg$$

Wrong

$$= 2mg.$$

If the velocity at lowest point is less than $\sqrt{5Rg}$ then we know that these body cannot complete the circle so to calculate the velocity & tension we will use MTR 2 & MTR 1 means we will put $T=0$ so θ will come out then we can get theg heigh $h(1-\cos\theta)$ & then wecan calculate velocity.



$$T = mg(3\cos\theta - 2) + \frac{mu^2}{R}$$

$$0 = \cancel{mg(3co}$$

$$= 3mg\cos\theta - 2mg + \frac{m4\cancel{R}g}{\cancel{R}}$$

$$= 3mg\cos\theta + 2mg$$

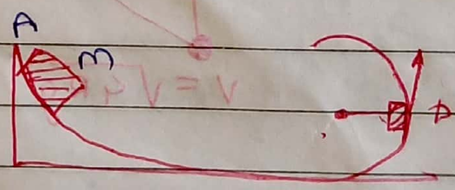
$$\cos\theta = \frac{-2mg}{3mg}$$

$$\cos\theta = -2/3$$

$$\text{height} = h(1-\cos\theta)$$
$$= R\left(1-\left(\frac{-2}{3}\right)\right)$$
$$= \frac{5R}{3}$$

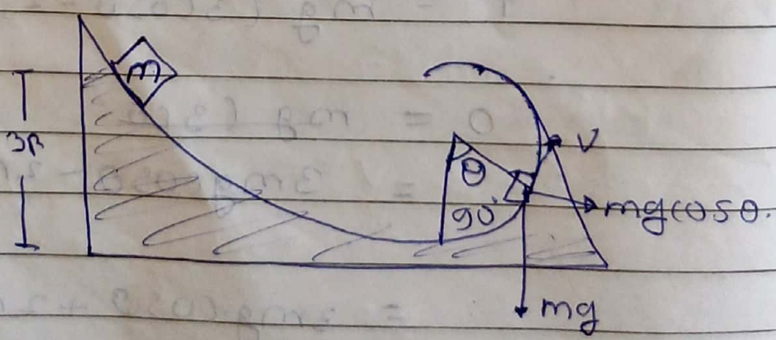
$$V = \sqrt{4Rg - 2g\,\tfrac{5}{3}R}$$
$$= \sqrt{4Rg - \frac{10gR}{3}}$$
$$= \cancel{2Rc}\sqrt{\frac{2Rg}{3}}$$

\# It a body is release from the point given in position as shown in fig then find how much force it will press the tract at the point P.



$V_p = ?$

$N = ?$



$$N - mg\cos\theta = \frac{mv^2}{R}$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v_p^2 + \cancel{m}gR(1-\cos\theta)$$

$$v_p = \sqrt{2gh - 2gr(1-\cos\theta}$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v_p^2 + \cancel{m}gR(1-\cos 90^\circ)$$

$$2gh = v_p^2 + gR$$

$$v_p = \sqrt{2gh - gR}$$

$$2g(3R) = v_p^2 + gR$$